Practical Design to Eurocode 2

The webinar will start at 12.30

(Any questions beforehand? - use ‘Questions’ on the GoTo Control Panel)

Course Outline

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Analysis

Lecture 4
14\textsuperscript{th} October 2015

Model Answers to

Week 3
Beam Exercise - Flexure & Shear

Nominal cover = 35 mm to each face

f_{ck} = 30MPa

Design the beam in flexure and shear

<table>
<thead>
<tr>
<th>Φ</th>
<th>Area, mm²</th>
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<tbody>
<tr>
<td>8</td>
<td>50</td>
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<tr>
<td>10</td>
<td>78.5</td>
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<tr>
<td>12</td>
<td>113</td>
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<td>201</td>
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<td>20</td>
<td>314</td>
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<td>25</td>
<td>491</td>
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<td>32</td>
<td>804</td>
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G_k = 10 kN/m, Q_k = 6.5 kN/m (Use EC0 eq. 6.10)

Lecture 4

Aide memoire

Example: ULS Combination of Actions

Partial Factors for Actions (ULS)

| q_k | 1.35 | (NA 2.2.3.3 and Table NA.4.11.2) |
| q_k | 1.35 | (NA 2.2.3.3 and Table NA.4.11.2) |

Beam Example 1 - Shear

Shear force, V_{Ed} = (90.35 x 0.2 + 705.8) kN

<table>
<thead>
<tr>
<th>V_{Ed}</th>
<th>90.35</th>
<th>705.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear stress, (\tau_{cr})</td>
<td>(\frac{V_{Ed}}{b_d h})</td>
<td></td>
</tr>
<tr>
<td>90.35</td>
<td>(\frac{90.35}{0.30 \times 450})</td>
<td></td>
</tr>
<tr>
<td>705.8</td>
<td>(\frac{705.8}{0.30 \times 450})</td>
<td></td>
</tr>
</tbody>
</table>

Design Flowchart
Solution - Flexure

ULS load per m = (10 x 1.35 + 6.5 x 1.5) = 23.25 kN/m

\[ M_{ult} = 23.25 \times 8^2 / 8 = 186 \text{ kNm} \]

\[ d = 450 - 35 - 10 - 32 / 2 = 389 \text{ mm} \]

\[ K = \frac{M}{bd^2 f_{ck}} = \frac{186 \times 10^4}{300 \times 389^2 \times 30} = 0.137 \quad K' = 0.208 \]

\[ K < K' \Rightarrow \text{No compression reinforcement required} \]

\[ z = \frac{d}{2} \left[ 1 + \sqrt{1 - 3.53K} \right] = \frac{389}{2} \left[ 1 + \sqrt{1 - 3.53 \times 0.137} \right] = 0.86 \times 389 = 334 \leq 0.95d \]

\[ A_s = \frac{M}{f_{yd} z} = \frac{186 \times 10^4}{435 \times 334} = 1280 \text{ mm}^2 \]

Provide 3 H25 (1470 mm²)

Solution - Shear

Shear force, \( V_{Ed} = 23.25 \times 8 / 2 = 93 \text{ kN} \)

Shear stress:

\[ V_{Ed} = V_{Ed} / (b_w \times 0.9d) = 93 \times 10^3 / (300 \times 0.9 \times 389) = 0.89 \text{ MPa} \]

\[ V_{Rd} = 3.64 \text{ MPa} \]

\[ V_{Rd} > V_{Ed} \quad \therefore \cot \theta = 2.5 \]

\[ A_{sw} / s = V_{Ed} b_w / (f_{ywd} \cot \theta) \]

\[ A_{sw} / s = 0.89 \times 300 \times 435 \times 2.5 \]

\[ A_{sw} / s = 0.24 \text{ mm} \]

Try H8 links with 2 legs, \( A_{sw} = 101 \text{ mm}^2 \)

\[ s < 101 / 0.24 = 420 \text{ mm} \]

Maximum spacing = 0.75 \( d = 0.75 \times 389 = 292 \text{ mm} \)

⇒ provide H8 links at 275 mm spacing
Analysis

Lecture 4
12th October 2017

Summary: Lecture 4

EN 1992-1-1: Section 5 Structural Analysis:
- Section 5.1 General
- Section 5.2 Geometric Imperfections
- Section 5.3 Idealisation
- Sections 5.4 & 5.5 Linear Elastic Analysis
- Section 5.6 Plastic Analysis
- Section 5.6.4 Strut & Tie
  - Worked Example Using Strut & Tie
- Sections 5.7, 5.9 to 5.11 - outline
- Section 5.8 Columns - separate lecture
- Exercises
Section 5.1 General

Structural Analysis (5.1.1)

- Common idealisations used:
  - linear elastic behaviour
  - linear elastic behaviour with limited redistribution
  - plastic behaviour
  - non-linear behaviour

- Local analyses are required where linear strain distribution is not valid:
  - In the vicinity of supports
  - Local to concentrated loads
  - In beam/column intersections
  - In anchorage zones
  - At changes in cross section
Soil/Structure Interaction (5.1.2)

- Where soil/structure interaction has a significant affect on the structure use EN 1997-1

Basement walls design:
- with/without ground water,
- partial load and material factors to EN1997 Combinations 1 and 2
- presumed backfilling occurs after GF slab (otherwise cantilever case!)
Soil/Structure Interaction (5.1.2)

- Where soil/structure interaction has a significant affect on the structure use EN 1997-1

- Simplifications (see Annex G) include:
  - flexible superstructure
  - rigid superstructure; settlements lie in a plane
  - foundation system or supporting ground assumed to be rigid

- Relative stiffness between the structural system and the ground > 0.5 indicates a rigid structural system

Second Order Effects (5.1.4)

- For buildings 2nd order effects may be ignored if they are less than 10% of the corresponding 1st order effects.
  (But of course you first need to know how big they are!)

- For 2nd order effects with axial loads, (columns), two alternative methods of analysis are permitted:
  - Method A based on nominal stiffnesses (5.8.7)
  - Method B based on nominal curvature (5.8.8) ✔
Section 5.2 Geometric Imperfections

Geometric Imperfections (5.2)

- Out-of-plumb imperfection is represented by an inclination, $\theta_i$
  where
  $\theta_i = \theta_0 \alpha_h \alpha_m$
  where $\theta_0 = 1/200$
  $\alpha_h = 2/\sqrt{l}$; $2/3 \leq \alpha_h \leq 1$
  $\alpha_m = \sqrt{0.5(1+1/m)}$
  $l$ is the height of member (m)
  $m$ is the number of vert. members

- Deviations in cross-section dimensions are normally taken into account in the material factors and should not be included in structural analysis

- Imperfections need not be considered for SLS
Geometric Imperfections (5.2)

The effect of geometric imperfections in isolated members may be accounted for in one of two different ways: either:

a) as an eccentricity, $e_i$, where

$$e_i = \frac{\theta_i l_0}{2}$$

So for walls and isolated columns $e_i = \frac{l_0}{400}$,

or

b) as a transverse force, $H_i$, where

$$H_i = \theta_i N \text{ for unbraced members}$$

$$H_i = 2\theta_i N \text{ for braced members}$$

Geometric Imperfections (5.2)

a) & b) (cont)

$e_i$ and $H_i$ in isolated members (e.g., columns)
Geometric Imperfections (5.2)

b) (cont)

$H_i$ in structural systems

- **Bracing System**
  \[ H_i = \theta_i (N_b - N_a) \]

- **Floor Diaphragm**
  \[ H_i = \theta_i (N_b + N_a)/2 \]

- **Roof**
  \[ H_i = \theta_i N_a \]

Partial factors for $H_i$

It is not clear how the notional force $H_i$ should be regarded, i.e. as a permanent action, a variable action, an accidental action.

However by inference if should be the same as for the constituent axial loads $N$, $N_{Ed}$, $N_a$ and/or $N_b$.

i.e. $\gamma_{Hi} = (1.35G_k + 1.5Q_k)/(G_k + Q_k)$

But TCC’s Worked Examples says “As $H_i$ derives mainly from permanent actions its resulting effects are considered as being a permanent action too.” and $\gamma_{Hi} = \gamma_G = 1.35$ was used.
Section 5.3 Idealisation

Idealisation of the structure (5.3)

- Beam: Span ≥ 3h otherwise it is a deep beam
- Slab: Minimum panel dimension ≥ 5h
  - One-way spanning
- Ribbed or waffle slabs need not be treated as discrete elements provided that:
  - rib spacing ≤ 1500mm
  - rib depth below flange ≤ 4b
  - flange depth ≥ 1/10 clear distance between ribs or 50mm
  - transverse ribs are provided with a clear spacing ≤ 10h
- Column: h ≤ 4b and L ≥ 3h otherwise it should be considered as a wall
Effective Flange Width
(5.3.2.1)

\[ b_{\text{eff}} = \sum b_{\text{eff},i} + b_w \leq b \]

where

\[ b_{\text{eff},i} = 0,2b_i + 0,1l_0 \leq 0,2l_0 \]

and \( b_{\text{eff},i} \leq b_i \)

\( l_0 \), is the distance between points of zero moment, viz:

\[ l_0 = 0,85b \]

\[ l_0 = 0,15(b_i + l_0) \]

\[ l_0 = 0,7b \]

\[ l_0 = 0,15b_i + l_0 \]

Effective Length of Beam or Slab
(5.3.2.2)

\[ l_{\text{eff}} = l_n + a_1 + a_2 \]

- The design moment and reaction for monolithic support should generally be taken as the greater of the elastic and redistributed values.
- Critical design moment usually at face of support.
  \( (\geq 0,65 \text{ the full fixed moment}) \).
- Permitted reduction, \( \Delta M_{\text{Ed}} = F_{\text{Ed.sup}}l/8 \)
Sections 5.4 & 5.5
Linear Elastic Analysis

Linear Elastic Analysis
(5.4)

- Linear elastic analysis may be used for both ULS and SLS
- Linear elastic analysis may be carried assuming:
  - uncracked sections (concrete section only)
  - linear stress-strain relationships
  - mean value of the modulus of elasticity
- For thermal deformation, settlement and shrinkage effects at ULS a reduced stiffness corresponding to cracked sections may be assumed.
Linear Elastic Analysis with Limited Redistribution (5.5)

- In continuous beams or slabs which are mainly subject to flexure and for which the ratio of adjacent spans is between 0.5 and 2

\[ \delta \geq 0.4 + (0.6 + 0.0014/E_{cu2})x_{u}/d \]

\[ \geq 0.7 \text{ for Class B and C reinforcement} \]

\[ \geq 0.8 \text{ for Class A reinforcement} \]

where:

- \( \delta \) is (redistributed moment)/(elastic moment)
- \( x_{u} \) is the neutral axis depth after redistribution

\[ z = d \left[ 1 + (1 - 3.53K)^{0.5} \right]/2 \]

\[ x = 2.5(d - z) \]

\[ \therefore x/d = 2.5\left(1 + (1 - 3.53K)^{0.5}\right)/2 \]
Redistribution Limits

\[ \delta = 1 - \frac{\text{redist}}{100} \]

for Class B & C Steel

for Class A Steel

Linear Elastic Analysis with Limited Redistribution (5.5)

- For column design, the elastic values from the frame analysis should be used (not the redistributed values).
Flat Slabs: Annex I

The division of Flat Slabs into Column strips and Middle strips is dealt with in Annex I, under Equivalent Frame Analysis.

As week 5

<table>
<thead>
<tr>
<th></th>
<th>Negative moments</th>
<th>Positive moments</th>
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<tr>
<td>Column Strip</td>
<td>60 - 80%</td>
<td>50 - 70%</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>40 - 20%</td>
<td>50 - 30%</td>
</tr>
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Note: Total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%.

5.6 Plastic Analysis
Plastic Analysis
(5.6)

- Plastic analysis may be used for ULS
- Plastic analysis requires ductility which is provided if:
  - $x_u/d \leq 0.25$ for $\leq C50/60$
    (or $x_u/d \leq 0.15$ for $\leq C55/67$) ($\equiv K = 0.10$)
    and
  - Class B or C reinforcement used
    and
  - $0.5 < M_{sppt} / M_{span} < 2.0$
- For higher strengths - check rotation capacity to 5.6.3.

Plastic Analysis: Yield Line

Yield Line Method
Based on the 'work method'

External energy $E$ expended by the displacement of loads

Internal energy $D$ dissipated by the yield lines rotating
Plastic Analysis : Yield Line & Flat Slabs

5.6.4 Strut and Tie
What is strut and tie?
Strut-and-tie models (STM) are trusses consisting of struts, ties and nodes.

a) Modelling
Imagine or draw stress paths which show the elastic flow of forces through the structure.

b) STM
Replace stress paths with polygons of forces to provide equilibrium.

Conventionally, struts are drawn as dashed lines, ties as full lines and nodes numbered.

What is strut and tie?

c) Design members
- Design ties
- Check nodes and struts (may be)

d) Iterate
Minimise strain energy (minimise length of ties)
What is strut and tie?

Strut and tie models are based on the lower bound theorem of plasticity which states that any distribution of stresses resisting an applied load is safe providing:

- Equilibrium is maintained
- and
- Stresses do not exceed “yield”

What is strut and tie?

In strut and tie models trusses are used with the following components:

- Struts (concrete)
- Ties (reinforcement)
- Nodes (intersections of struts and ties)

Eurocode 2 gives guidance for each of these.

In principle - where non-linear strain distribution exists, strut and tie models may be used. e.g

- Supports
- Concentrated loads
- Openings
What is Strut and Tie?

A structure can be divided into:

- B (or beam or Bernoulli) regions in which plane sections remain plane and design is based on ‘normal’ beam theory,

- D (or disturbed) regions in which plane sections do not remain plane; so ‘normal’ beam theory may be considered inappropriate and Strut & Tie may be used.

Poll: At failure, what is $P_2 / P_1$?

Please answer:

a) $P_2 = 0.5 P_1$

b) $P_2 = 0.66 P_1$

c) $P_2 = 0.75 P_1$

d) $P_2 = P_1$

e) $P_2 = 1.33 P_1$

f) $P_2 = 2.0 P_1$
Struts

Cross-sectional dimensions of the strut are determined by dimensions of the nodes and assumptions made there.

Usually its thickness x dimension ‘a’ (see Figures)

The stress in struts is rarely critical but the stress where struts abut nodes is (see later).

However . . . . .

6.5.2 Struts

Where there is no transverse tension

\[ \sigma_{Rd,\text{max}} = f_{cd} \]
\[ = 0.85 f_{ck} / 1.5 \]
\[ = 0.57 f_{ck} \]

Otherwise, where there is transverse tension

\[ \sigma_{Rd,\text{max}} = 0.6 \nu' f_{cd} \]

Where:

\[ \nu' = 1 - f_{ck} / 250 \]

\[ \sigma_{Rd,\text{max}} = 0.6 \times (1 - f_{ck} / 250) \times 1.0 \times f_{ck} / 1.5 \]
\[ = 0.4 \times (1 - f_{ck} / 250) f_{ck} \]
Bi-axial Strength of Concrete

Axial stress $\sigma_x$

Transverse tension

Loss in axial strength due to transverse tension

Transverse stress, $\sigma_y$ or $\sigma_z$

6.5.3 Struts & Discontinuities

Areas of non-linear strain distribution are referred to as “discontinuities”

Partial discontinuity

Full discontinuity

Curved compression trajectories lead to tensile forces

Curved compression trajectories lead to tensile forces

$b_v = b$

$b_v = 0.5h + 0.65a; \ a \leq h$
Partial discontinuity

Tension in the reinforcement is $T$

When $b \leq H/2$

$$T = \frac{1}{4} \left[ \left( b - a \right) / b \right] F$$

Full discontinuity

When $b > H/2$

$$T = \frac{1}{4} \left( 1 - 0.7a / h \right) F$$

Reinforcement ties to resist the transverse force $T$ may be “discrete” or can be “smeared” over the length of tension zone arising from the compression stress trajectories.

If required such transverse reinforcement should be placed between $0.4h$ and $1.0h$ from the loaded surface.
6.5.4 Nodes

Nodes are typically classified as:

- CCC - Three compressive struts
- CCT - Two compressive struts and one tie
- CTT - One compressive strut and two ties

The maximum stress at the edge of the node:

$$\sigma_{Rd,max} = k_1 \nu' f_{cd}$$

Where:

- $$k_1 = 1.0$$
- $$\nu' = 1 - f_{ck}/250$$

$$\sigma_{Rd,max} = (1 - f_{ck}/250) \times 0.85 \times f_{ck} / 1.5$$

$$= 0.57 (1 - f_{ck}/250) f_{ck}$$

NB Hydrostatic pressures: the stresses $$\sigma_{c0}, \sigma_{Rd,1}, \sigma_{Rd,2} \text{ & } \sigma_{Rd,2}$$ etc are all the same.
CCT nodes

The maximum compressive stress is:

$$\sigma_{Rd,max} = k_2 \cdot \nu' f_{cd}$$

Where:

- $k_2 = 0.85$
- $\nu' = 1 - \frac{f_{ck}}{250}$

$$\sigma_{Rd,max} = 0.85 \cdot \left(1 - \frac{f_{ck}}{250}\right) x 0.85 x \frac{f_{ck}}{1.5} = 0.48 \left(1 - \frac{f_{ck}}{250}\right) f_{ck}$$

(stresses in the two struts should be equal)

CTT nodes

The maximum compressive stress is:

$$\sigma_{Rd,max} = k_3 \cdot \nu' f_{cd}$$

Where:

- $k_3 = 0.75$
- $\nu' = 1 - \frac{f_{ck}}{250}$

$$\sigma_{Rd,max} = 0.75 \cdot \left(1 - \frac{f_{ck}}{250}\right) x 0.85 x \frac{f_{ck}}{1.5} = 0.43 \left(1 - \frac{f_{ck}}{250}\right) f_{ck}$$
Ties

Design strength, $f_{yd} = f_{yk}/1.15$

Reinforcement should be anchored into nodes
The anchorage may start as the bar enters the strut

Construction of an STM for a deep beam using the load path method

Usually taken as a maximum of 1:2
Strut & Tie Models

Similarity (single point load)

(a) H = L

(b) H = 1.5 L

(c) H = 2.0 L

(d) H = 2.5 L

Worked Example
Pile-Cap Using S&T
Pile-cap example

Using a strut and tie model, what tension reinforcement is required for a pile cap supporting a 500 mm square column carrying 2500 kN (ULS), and itself supported by two-piles of 600 diam @ 1800 cc.

\[ f_{ck} = 30 \text{ MPa} \]

**Breadth = 900 mm**

\[
\text{Width of strut}^* = \frac{250}{\cos 34.7^\circ} = 304\text{ mm}
\]

\[
\text{Force per strut} = \frac{1250}{\cos 34.7^\circ} = 1520 \text{ kN}
\]

\[
\text{Force in tie} = 1250 \tan 34.7^\circ = 866 \text{ kN}
\]

* Conventional but simplistic - see later
**Pile-cap example**

Area of steel required:

\[
A_s \geq 866 \times 10^3 / 435 \\
\geq 1991 \text{ mm}^2
\]

Use 5 H25s (2455 mm²)

Usually that is probably as far as you would go. But for the sake of completeness and the exercise you will be undertaking we will continue:

**Pile-cap example**

Check forces in truss

Stress in strut (top - under half of column)

\[
\sigma_{Ed} = 10.0 \text{ MPa}
\]

Strength of strut:

\[
\sigma_{Rd,max} = 0.4 \left(1 - \frac{f_{ck}}{250}\right) f_{ck}
\]

= 10.6 MPa

OK
Pile-cap example

Nodes: top

From before
\[ \sigma_{Ed,2} = 10.0 \text{ MPa (from above)} \]
\[ \sigma_{Ed,3} = 10.0 \text{ MPa (as above)} \]
\[ \sigma_{Ed,1} = \frac{2500 \times 10^3}{(500^2)} = 10.0 \text{ MPa} \]

\[ \sigma_{Rd,max} \text{ (for CCC node)} = 0.57 \left(1 - \frac{f_{ck}}{250}\right) \]
\[ f_{ck} = 15.0 \text{ MPa} \]

\[ \sigma_{Ed,2} \]
\[ \sigma_{Ed,1} \]

Pile-cap example

Nodes: bottom (as a check)

Strut above
Width of strut* = \( \frac{600}{\cos 34.7°} \)
= 730 mm

Stress in strut (bottom as an ellipse)
\[ \sigma_{Ed,2} = \frac{1520 \times 10^3}{(600 \times 730 \times \pi/4)} = 4.4 \text{ MPa} \]
\[ \sigma_{Ed,1} = \frac{1250 \times 10^3}{(\pi \times 300^2)} = 4.4 \text{ MPa} \]

\[ \sigma_{Rd,max} \text{ (for CCT node)} = 0.48 \left(1 - \frac{f_{ck}}{250}\right) \]
\[ f_{ck} = 12.7 \text{ MPa} \text{ OK} \]

* Conventional but simplistic - see later
Pile-cap example

Detailing
Detailed checks are also required for the following:
- Small piles
- Determine local tie steel across struts (if req’d)
- Detailing of reinforcement anchorage (large radius bends may be required)

Using S&T, anchorage starts from here (100%)

Strut dimensions

Re* previous statement that calculated strut dimension as 304 mm were “Conventional but simplistic - see later”

For the CCT node:
- Not used in previous calc.
- Hence struts themselves rarely critical.

Similarly for the CCC node

Notice too that there is a vertical distance between the centroid of forces and the surface, which is often ignored
Comparison: Pile-cap example

Compare previously designed pile cap using bending theory

\[ M_{Ed} = 2500 \times 1.800 / 4 = 1125 \text{ kNm} \]

Assume:
25 mm \( \phi \) for tension reinforcement
12 mm link

\[
\begin{align*}
    d &= h - c_{\text{nom}} - \phi_{\text{link}} - 0.5\phi \\
    &= 1400 - 75 - 12 - 13 \\
    &= 1300 \text{ mm}
\end{align*}
\]

\[
A_s = \frac{1125 \times 10^6}{435 \times 1270} = 2036 \text{ mm}^2
\]

Use 5 H25 (2454 mm²)

c.f. using S&T 1991 mm² req'd and 5H25 provided
Sections 5.7 - 5.10: outline

Other Items re. Analysis

5.7 Non-linear analysis
May be used for ULS and SLS provided equilibrium and compatibility are satisfied and sections can withstand inelastic deformations.

5.8 Analysis of second order effects with axial load.
Slenderness and 2nd order moments - dealt with in Columns lecture, viz:

5.9 Lateral instability of slender beams
Limits on h/b in slender rectangular beams

5.10 Prestressed members and structures.
Max stressing forces, max concrete stresses, prestress force, losses, effects of prestressing

5.11 Particular structural members
Flat slabs are flat slabs and shear walls are shear walls (!!)
Design Methods

**Elastic methods** – e.g. moment distribution, continuous beam, sub-frame, plane frame, etc – all acceptable.

**Plastic methods** – yield line, Hillerborg - acceptable

**Finite Element Methods** - elastic, elasto-plastic, non-linear etc - acceptable

Common pitfalls:
- Using long term E-values (typically 1/2 to 1/3 short term value)
- Not using cracked section properties (typically 1/2 gross properties) by adjusting E-value to suit
- Therefore appropriate E-values are usually 4 to 8 kN/mm²
- $M_{\text{max}}$ (and column transfer moments)
- Check punching

---

Exercises

Lecture 4
Design Exercise: Pile Cap Using S&T

Using a strut and tie model, what tension reinforcement is required for a pile cap supporting a 650 mm square column carrying 4000 kN (ULS), and itself supported by two-piles of 750 mm diameter?

Are the node stresses OK?

\( f_{ck} = 30 \, \text{MPa} \)

Breadth = 1050 mm

\( f_{ck} = 30 \, \text{MPa} \)

If there is time:
Design \( A_{reqd} \) using beam theory

Angle of strut = \( \tan^{-1}(\_\_\_\_/\_\_\_) \)

= ___

Width of strut (\( ? \)) = \_\_\_/cos____

= ____ mm

Force per strut = \_\_\_/cos____

= ____ kN

Force in tie = ____ tan____

= ____ kN

Design Exercise: Pile Cap Using S&T (pro forma)
Design Exercise: Pile Cap Using S&T (pro forma)

Check forces in truss
Stress in strut (top) = \( \frac{\sigma_{Ed}}{10^3} \times \frac{1}{(\text{top \times 650})} \)

\( \sigma_{Ed} \) = ____ MPa

Strength of strut:
\( \sigma_{Rd,\text{max}} = 0.4 \left( 1 - \frac{f_{ck}}{250} \right) f_{ck} \)

\( f_{ck} \) = ____ MPa

Area of steel required:
\( A_s \) \( \geq \) \( \frac{10^3 \times 435}{10^3} \)

\( \geq \) ______ mm²

Use _____ H ______

Design Exercise: Pile Cap Using S&T (pro forma)

Nodes: bottom

Above pile
\( \sigma_{Ed,1} = \frac{2000 \times 10^3}{2 \pi} \times 10^2 \)

\( \sigma_{Ed,1} \) = ____ MPa

\( \sigma_{Ed,2} \) = ____ MPa (say as above)

\( \sigma_{Rd,\text{max}} = 0.48 \left( 1 - \frac{f_{ck}}{250} \right) f_{ck} \)

\( f_{ck} \) = ____ MPa
Design Exercise:
Pile Cap Using S&T (pro forma)

Nodes: top

From above
\[ \sigma_{Ed,1} = \frac{4000 \times 10^3}{650^2} \]
\[ = \underline{\text{MPa}} \]
\[ \sigma_{Ed,2} = \underline{\text{MPa}} \]
\[ \sigma_{Ed,3} = \underline{\text{MPa}} \]

\[ \sigma_{Rd,max} = 0.57 \left(1 - \frac{f_{ck}}{250}\right) f_{ck} \]
\[ = \underline{\text{MPa}} \]
Summary: End of Lecture 4

EN 1992-1-1: Section 5 Structural Analysis:
- Section 5.1 General
- Section 5.2 Geometric Imperfections
- Section 5.3 Idealisation
- Sections 5.4 & 5.5 Linear Elastic Analysis
- Section 5.6 Plastic Analysis
- Section 5.6.4 Strut & Tie
- Worked Example Using Strut & Tie
- Sections 5.7, 5.9 to 5.11 - outline
- Section 5.8 Columns - separate lecture
- Exercises

Course Outline

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