Slabs and Flat Slabs

Lecture 5
19\textsuperscript{th} October 2017

Contents - Lecture 5

- Designing for shear in slabs - including punching shear
- Detailing - Solid slabs
- Flat Slab Design - includes flexure worked example
- Exercise - Punching shear
Designing for shear in slabs

- When shear reinforcement is not required - e.g. usually one and two-way spanning slabs
- Punching shear - e.g. flat slabs and pad foundations

Shear

There are three approaches to designing for shear:
- When shear reinforcement is not required e.g. usually slabs
- When shear reinforcement is required e.g. Beams, see Lecture 3
- Punching shear requirements e.g. flat slabs

The maximum shear strength in the UK should not exceed that of class C50/60 concrete
Shear resistance without shear reinforcement

\[ V_{Rd,c} = [0.12k(100 \, \rho \, f_{ck})^{1/3} + 0.15\sigma_{cp}] \, b_w d \]  \hspace{1cm} (6.2.a)

with a minimum of

\[ V_{Rd,c} = (0.035k^{3/2}f_{ck}^{1/2} + 0.15 \, \sigma_{cp}) \, b_w d \]  \hspace{1cm} (6.2.b)

where:

- \( k = 1 + \sqrt{200/d} \leq 2.0 \)
- \( \rho = A_{sl}/b_w d \leq 0.02 \)
- \( A_{sl} = \) area of the tensile reinforcement,
- \( b_w = \) smallest width of the cross-section in the tensile area [mm]
- \( \sigma_{cp} = N_{Ed}/A_c < 0.2 \, f_{ck} \) [MPa] Compression +ve
- \( N_{Ed} = \) axial force in the cross-section due to loading or pre-stressing [in N]
- \( A_c = \) area of concrete cross section [mm²]
### Shear - $\nu_{Rd,c}$

**Concise Table 7.1 or 15.6**

<table>
<thead>
<tr>
<th>$A_s$ ($bd$) %</th>
<th>Effective depth, $d$ (mm)</th>
<th>$\nu_{Rd,c}$ resistance of members without shear reinforcement, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>≤200</td>
<td>225</td>
</tr>
<tr>
<td>0.25</td>
<td>0.54</td>
<td>0.52</td>
</tr>
<tr>
<td>0.50</td>
<td>0.59</td>
<td>0.57</td>
</tr>
<tr>
<td>0.75</td>
<td>0.69</td>
<td>0.66</td>
</tr>
<tr>
<td>1.00</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>1.25</td>
<td>0.79</td>
<td>0.78</td>
</tr>
<tr>
<td>1.50</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>1.75</td>
<td>0.90</td>
<td>0.87</td>
</tr>
<tr>
<td>2.00</td>
<td>0.94</td>
<td>0.91</td>
</tr>
</tbody>
</table>

$k$<br>
2.00 | 1.94 | 1.89 | 1.85 | 1.82 | 1.76 | 1.71 | 1.67 | 1.63 | 1.58 | 1.52

Table derived from: $\nu_{Rd,c} = 0.12 k (100 \rho_I f_{ck})^{1/3} \geq 0.035 k^{1.5} f_{ck}^{0.5}$ where $k = 1 + \sqrt{200/d} \leq 2$ and $\rho_I = A_s/(bd) \leq 0.02$

Note: This table has been prepared for $f_{ck} = 30$. Where $\rho_I$ exceeds 0.40%, the following factors may be used:

<table>
<thead>
<tr>
<th>$f_{ck}$</th>
<th>25</th>
<th>28</th>
<th>32</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>factor</td>
<td>0.94</td>
<td>0.98</td>
<td>1.02</td>
<td>1.05</td>
<td>1.10</td>
<td>1.14</td>
<td>1.19</td>
</tr>
</tbody>
</table>

### Shear in Slabs

Most slabs do not require shear reinforcement.

∴ Check $V_{Ed} < V_{Rd,c}$

Where $V_{Rd,c}$ is shear resistance of members without reinforcement

$$V_{Rd,c} = 0.12 k (100 \rho_I f_{ck})^{1/3} \geq 0.035 k^{1.5} f_{ck}^{0.5}$$

Where $V_{Ed} > V_{Rd,c}$, shear reinforcement is required and the strut inclination method should be used.

How-to Compendium p21
Punching shear

Punching shear symbols

\[ u_i = \text{i}^{\text{th}} \text{ perimeter} \]
\[ u_1 = \text{basic control perimeter at } 2d \]
\[ u_{1*} = \text{reduced basic control perimeter} \]
\[ u_0 = \text{column perimeter} \]
\[ d = \text{average effective depth} \]
\[ k = \text{coeff. depending on column shape - see Table 6.1} \]
\[ W_1 = \text{a shear distribution factor - see 6.4.3(3)} \]

Punching Shear

EC2: Cl. 6.4

Punching shear does not use the Variable Strut inclination method and is similar to BS 8110 methods

- The basic control perimeter is set at 2d from the loaded area
- The shape of control perimeters have rounded corners

- Where shear reinforcement is required the shear resistance is the sum of the concrete and shear reinforcement resistances.
Punching Shear

EC2: Cl. 6.4.3 & 6.4.4

6.4.3 (2)

(b) Punching shear reinforcement is not necessary if:

\[ V_{Ed} \leq V_{Rd,c} \]

When calculating \( V_{Rd,c} \):

6.4.4 (1)

\( \rho = \sqrt{\rho_y \cdot \rho_z} \leq 0.02 \)

\( \rho_y, \rho_z \) relate to the bonded tension steel in \( y \)- and \( z \)-directions respectively. The values \( \rho_y \) and \( \rho_z \) should be calculated as mean values taking into account a slab width equal to the column width plus 3\( d \) each side.

Punching Shear

The applied shear stress should be taken as:

\[ V_{Ed} = \beta \frac{V_{Ed}}{u_1 d} \]

where \( \beta = 1 + k \frac{M_{Ed}}{V_{Ed}} u_1 / W_1 \)

For structures where:

- lateral stability does not depend on frame action
- adjacent spans do not differ by more than 25%

the approximate values for \( \beta \) shown may be used:
For a rectangular internal column with biaxial bending the following simplification may be used:

$$\beta = 1 + 1.8 \left( \left( \frac{e_y}{b_y} \right)^2 + \left( \frac{e_z}{b_z} \right)^2 \right)^{0.5}$$

where $b_y$ and $b_z$ are the dimensions of the control perimeter.

For other situations there is plenty of guidance on determining $\beta$ given in Cl 6.4.3 of the Code.

### Punching shear control perimeters

**Basic perimeter, $u_1$**

**EC2: Cl. 6.4.2**

Near to an edge

**Concise: Figure 8.4**

Near to an opening

**Concise: Figure 8.6**
The outer control perimeter at which shear reinforcement is not required, should be calculated from:

\[ u_{\text{out,ef}} = \beta V_{\text{Ed}} / (\nu_{\text{Rd,c}} d) \]

\[ v_{\text{Ed}} = \beta V_{\text{Ed}} / u_1 d \]

The outermost perimeter of shear reinforcement should be placed at a distance not greater than \( kd \) (\( k = 1.5 \)) within the outer control perimeter.
Punching Shear Reinforcement

EC 2: Cl. 6.4.5, Equ 6.52 Concise: 8.5

Where shear reinforcement is required it should be calculated in accordance with the following expression:

\[ v_{Rd,cs} = 0.75 v_{Rd,c} + 1.5 \left( \frac{d}{s_r} \right) A_{sw} f_{ywd,ef} \left( 1/\left( u_1 d \right) \right) \sin \alpha \]  

(6.52)

- \( A_{sw} \) = area of shear reinforcement in each perimeter around the column
- \( s_r \) = radial spacing of layers of shear reinforcement
- \( \alpha \) = angle between the shear reinforcement and the plane of slab
- \( f_{ywd,ef} \) = effective design strength of the punching shear reinforcement, \( f_{ywd,ef} = 250 + 0.25 d \leq f_y \) (MPa.)
- \( d \) = mean effective depth of the slabs (mm)

Max. shear stress at column face,

\[ V_{Ed} = \frac{\beta V_{Ed}}{u_0 d} \leq V_{Rd,max} = 0.5 \nu f_{cd} \]

EC2 Equ 6.53

Punching Shear Reinforcement

EC 2: Cl. 6.4.5 (3), Equ 6.53 Concise: 8.6

Max. shear stress at column face, the \( u_0 \) perimeter

\[ V_{Ed} = \frac{\beta V_{Ed}}{u_0 d} \leq V_{Rd,max} = 0.5 \nu f_{cd} \]

where

\[ \beta = \text{factor dealing with eccentricity (see )} \]

\[ V_{Ed} = \text{applied shear force} \]

\[ d = \text{mean effective depth} \]

\[ u_0 = 2(c_1 + c_2) \text{ for interior columns} \]

\[ = 2c_2 + 3d \leq 2c_2 + 2c_1, \text{ for edge columns} \]

\[ = 3d \leq 2c_2 + 2c_1, \text{ for corner columns} \]

where

\[ c_1 = \text{column depth} \]

\[ c_2 = \text{column width} \]

\( c_1 \) and \( c_2 \) are illustrated in Concise Figure 8.5
Punching Shear Reinforcement

Check $v_{Ed} \leq 2 v_{Rdc}$ at basic control perimeter (NA check)

Note: UK NA says ‘first’ control perimeter, but the paper* on which this guidance is based says ‘basic’ control perimeter

The minimum area of a link leg (or equivalent), $A_{sw,\text{min}}$, is given by the following expression:

$$A_{sw,\text{min}} \geq \frac{(1.5 \sin \alpha + \cos \alpha) / (s_i s_j)}{(0.08 \sqrt{(f_{ck})}) / f_{yk}} \quad \text{EC2 equ 9.11}$$

$$A_{sw,\text{min}} \geq (0.053 s_i s_j \sqrt{(f_{ck})}) / f_{yk} \quad \text{For vertical links}$$


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Punching shear
Worked example

From Worked Examples to EC2: Volume 1
Example 3.4.10
Punching shear at column C2

**400 mm Square Column**

**300 mm flat slab C30/37 concrete**

**Design information**

- At C2 the ultimate column reaction is 1204.8 kN
- Effective depths are 260mm & 240mm
- Reinforcement: $\rho_{ly} = 0.0085$, $\rho_{lz} = 0.0048$

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Punching shear

**A few definitions:**

- $U_0$
- $U_1$
- $U_{\text{out}}$
- Outer control perimeter requiring shear reinforcement, $U_{\text{ext}}$
- Outer control perimeter not requiring shear reinforcement, $U_{\text{int}}$
- $2d$
- $1.5d$
- $0.5d$
- $0.75d$
Solution

1. Check shear at the perimeter of the column
\[ V_{Ed} = \beta V_{Ed} / (u_0 d) < V_{Rd, max} \]
\[ \beta = 1.15 \]
\[ u_0 = 4 \times 400 = 1600 \text{ mm} \]
\[ d = (260 + 240)/2 = 250 \text{ mm} \]
\[ V_{Ed} = 1.15 \times 1204.8 \times 1000 / (1600 \times 250) = 3.46 \text{ MPa} \]
\[ V_{Rd, max} = 0.5 \nu f_{cd} \]
\[ = 0.5 \times 0.6(1 \cdot f_{ck}/250) \times \alpha_{cc} f_{ck}/\gamma_m \]
\[ = 0.5 \times 0.6(1 \cdot 30/250) \times 1.0 \times 30 / 1.5 = 5.28 \text{ MPa} \]
\[ V_{Ed} < V_{Rd, max} \quad \text{...OK} \]

Solution

2. Check shear at \( u_1 \), the basic control perimeter
\[ V_{Ed} = \beta V_{Ed} / (u_1 d) < V_{Rd, c} \]
\[ \beta = \text{as before} \]
\[ u_1 = 2(c_z + c_y) + 2\pi \times 2d \]
\[ = 2(400 + 400) + 2\pi \times 2 \times 250 = 4742 \text{ mm} \]
\[ V_{Ed} = 1.15 \times 1204.8 \times 1000 / (4742 \times 250) = 1.17 \text{ MPa} \]
\[ V_{Rd, c} = 0.12 k(100/\rho f_{ck})^{1/3} \]
\[ k = 1 + (200/d)^{1/2} = 1 + (200/250)^{1/2} = 1.89 \]
\[ \rho_{\text{c}} = (\rho_{\text{ly}} \rho_{\text{lx}})^{1/2} = (0.0085 \times 0.0048)^{1/2} = 0.0064 \]
\[ V_{Rd, c} = 0.12 \times 1.89(100 \times 0.0064 \times 30)^{1/3} = 0.61 \text{ MPa} \]
\[ V_{Ed} > V_{Rd, c} \quad 1.17 \text{ MPa} > 0.61 \text{ MPa} \quad \text{... Therefore punching shear reinf. required} \]

2a. NA check:
\[ V_{Ed} \leq 2V_{Rd, c} \text{ at basic control perimeter} \]
\[ 1.17 \text{ MPa} \leq 2 \times 0.61 \text{ MPa} = 1.22 \text{ MPa} \quad \text{OK} \]
Solution

3. Perimeter at which punching shear no longer required

\[ u_{out} = \beta V_{Ed} / (d v_{Rd,c}) \]
\[ = 1.15 \times 1204.8 \times 1000 / (250 \times 0.61) \]
\[ = 9085 \text{ mm} \]

Rearrange:

\[ u_{out} = 2(c_x + c_y) + 2\pi r_{out} \]
\[ r_{out} = (u_{out} - 2(c_x + c_y)) / (2\pi) \]
\[ = (9085 - 1600) / (2\pi) = 1191 \text{ mm} \]

Position of outer perimeter of reinforcement from column face:

\[ r = 1191 - 1.5 \times 250 = 816 \text{ mm} \]

Maximum radial spacing of reinforcement:

\[ s_{r,max} = 0.75 \times 250 = 187 \text{ mm, say 175 mm} \]

Solution

4. Area of reinforcement

\[ A_{sw} \geq (V_{Ed} - 0.75V_{Rd,c} s_{t}) s_{i} / (1.5 f_{wld,ef}) \]
\[ f_{wld,ef} = (250 + 0.25d) = 312 \text{ MPa} \]

\[ A_{sw} \geq (1.17 - 0.75 \times 0.61) \times 175 \times 4741 / (1.5 \times 312) \]
\[ \geq 1263 \text{ mm}^2 / \text{perim.} \]

1H10 is 78.5 mm\(^2\) dia.

16H10 = 1256 mm\(^2\) / perim

Minimum area of a link leg:

\[ A_{sw,\text{min}} \geq (0.053 s_t s_i \sqrt{f_{ck}}) / f_{yk} = 0.053 \times 175 \times 350 \times 30 / 500 \]
\[ \geq 36 \text{ mm}^2 \]

could use H8s (50 mm\(^2\)) but would need 26 per perimeter So use H10s (same price as H8s!)
Solution

Detailing - Solid slabs
**Detailing - Solid slabs**

**EC2: Cl. 9.3**

Rules for one-way and two-way solid slabs

- Generally: as for beams.
- Where partial fixity exists, but not taken into account in design:
  - Internal supports: \( A_{s,\text{top}} \geq 0.25A_s \) for \( M_{\text{max}} \) in adjacent span
  - End supports: \( A_{s,\text{top}} \geq 0.15A_s \) for \( M_{\text{max}} \) in adjacent span
- This top reinforcement should extend \( \geq 0.2 \) adjacent span
- Reinforcement at free edges should include ‘u’ bars and longitudinal bars

![Diagram of slab](image)

- Secondary reinforcement 20% of principal reinforcement in one-way slabs

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**Flat Slab Design**
Flat Slab Design - Contents

Flat slabs - Introduction

EC2 particular rules for flat slabs

Initial sizing

Analysis methods - BM's and Shear Force

Design constraints
  - Punching shear
  - Deflection
  - Moment transfer from slab to column

Flat Slabs - Introduction

What are flat slabs?

- Solid concrete floors of constant thickness
- They have flat soffits
Flat Slabs - Introduction

- Column Head
- Drop Panel
- Waffle Slab

VOIED SLABS

1. COBIAX
2. BUBBLEDECK
Poll Q1:
Shear resistance of a beam section

What is the shear resistance governed by the crushing of compression struts?

a. $V_{Ed}$
b. $V_{Rd,c}$
c. $V_{Rd,max}$
d. $V_{Rd,s}$

Particular rules for flat slabs

EC2 sections relevant to Flat Slabs:

- Section 6 Ultimate Limit States
  - Cl 6.4 Punching (shear) & PD 6687 cl 2.16, 2.17 & 2.1.8
- Section 9 Detailing of members and particular rules
  - Cl 9.4 Flat slabs
    - 9.4.1 Slab at internal columns
    - 9.4.2 Slab at edge and corner columns
    - 9.4.3 Punching shear reinforcement
- Annex I (Informative) Analysis of flat slabs and shear walls
  - I.1 Flat Slabs
    - I.1.1 General
    - I.1.2 Equivalent frame analysis
    - I.1.3 Irregular column layout

The Concrete Society, Technical Report 64 - Guide to the Design and Construction of Reinforced Concrete Flat Slabs
Particular rules for flat slabs
Distribution of moments

EC2: Figure I.1

Concise Figure 5.11

Column strip

Middle strip

Column strip

Column strip

Middle strip

Column strip

Distribution of moments

EC2: Table I.1

Concise: Table 5.2

<table>
<thead>
<tr>
<th>Location</th>
<th>Negative moments</th>
<th>Positive moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column strip</td>
<td>60% – 80%</td>
<td>50% – 70%</td>
</tr>
<tr>
<td>Middle strip</td>
<td>40% – 20%</td>
<td>50% – 30%</td>
</tr>
</tbody>
</table>

Notes
- The total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%.
- The distribution of design moments given in BS 8110 (column strip: hogging 75%, sagging 55%; middle strip: hogging 25%, sagging 45%) may be used.
Particular rules for flat slabs

EC2: Cl. 9.4
Concise: 12.4.1

• Arrangement of reinforcement should reflect behaviour under working conditions.

• At internal columns $0.5A_t$ should be placed in a width $= 0.25 \times \text{panel width}$.

• At least two bottom bars should pass through internal columns in each orthogonal directions.

Particular rules for flat slabs

EC2: Figure 9.9, I.1.2(5)
Concise Figure 5.12

• Design reinforcement at edge and corner reinforcement should be placed within $b_e$

• The maximum moment that can be transferred from the slab to the column should be limited to $0.17b_ed^2f_{ck}$
Moment transfer

Edge and corner columns have limited capacity to transfer moments from slab – redistribution may be necessary

Rebar arrangement
Figure 47

Initial sizing

3 methods:

1. Simple span to depth table
2. Use Economic Concrete Frame Elements

<table>
<thead>
<tr>
<th>Imposed Load, Qa (kN/m²)</th>
<th>2.5</th>
<th>5</th>
<th>7.5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple Span</td>
<td>28</td>
<td>26</td>
<td>25</td>
<td>23</td>
</tr>
</tbody>
</table>
Initial sizing

3 methods:

1. Simple span to depth table
2. Use Economic Concrete Frame Elements
3. Use Concept.xls
Initial sizing

**Analysis Methods**

- Elastic Plane Frame - Equivalent Frame Method, Annex I
- Tabular Method - Equivalent Frame Method, Annex I
- Yield Line
  - Plastic method of design
- Finite Element Analysis
  - Elastic method
  - Elasto plastic
Analysis Methods

Elastic Plane Frame - Equivalent Frame Method, Annex I

- Apply in both directions - Y and Z
- Method of Analysis for Bending Moments & SF’s
- Equivalent Frame - the Beams are the Slab width
- $K_{\text{slab}}$ = use full panel width for vertical loads.
- $K_{\text{slab}}$ = use 40% panel width for horizontal loads. Annex I.1.2.(1)

Load cases

NA – can use single load case provided:
- Variable load $\leq$ 1.25 x Permanent load
- Variable load $\leq$ 5.0 kN/m²

Condition of using single load case is that Support BM’s should be reduced by 20% except at cantilever supports

Limitation of negative moments, $N_1$ and $N_2$
Analysis Methods

TR 64 - Figure 14
Reduction in maximum hogging moment at columns

Analysis Methods - Equi Frame

Distribution of Design Bending Moments, Annex I

<table>
<thead>
<tr>
<th>Table I.1</th>
<th>Column Strip</th>
<th>Middle Strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative</td>
<td>60 - 80%</td>
<td>40 - 20%</td>
</tr>
<tr>
<td>Positive</td>
<td>50 - 70%</td>
<td>50 - 30%</td>
</tr>
</tbody>
</table>

$A_e = \text{Reinforcement area to resist full negative moment. Cl 9.4.1}$
Analysis Methods - Equi Frame

Distribution of Design Bending Moments - Example

Table I.1  Column Strip  Middle Strip
Negative  75%  25%

$A_r$ = Reinforcement area to resist full negative moment. Cl 9.4.1

= 1600 mm$^2$

Column strip = 1200 mm$^2$  Middle strip = 400 mm$^2$

Equivalent frame method
Equivalent frame method

• Computer software normally used to assess bending moments and shear forces
• Design for full load in both directions
• RC spreadsheet TCC33.xls will carry out the analysis and design
### Analysis Methods - Tabular Method

e.g. use coefficients from Concise Tables 15.2 to determine bending moments and shear forces. \( BM = \text{coeff} \times n \times \text{span}^2 \) \( SF = \text{coeff} \times n \times \text{span} \)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Location</th>
<th>Table 15.2: Coefficients for use with one-way spanning slabs to Eurocode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End support/slab connection</td>
<td>Internal supports and spans</td>
</tr>
<tr>
<td></td>
<td>Outer support</td>
<td>Near middle of end span</td>
</tr>
<tr>
<td>Moment</td>
<td>0.0</td>
<td>0.086</td>
</tr>
<tr>
<td>Shear</td>
<td>0.40</td>
<td>0.40</td>
</tr>
</tbody>
</table>

- Design for **full** load in **both** directions
- Frame lateral stability must not be dependent on slab-col connections
- There must be at least three approx equal spans.
- Note: No column BM’s given in table.

### Analysis Methods - Tabular Method

A little more accurate:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Location</th>
<th>Table 15.3: Coefficients for use with beams (and one-way spanning slabs) to Eurocode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outer support</td>
<td>Near middle of end span</td>
</tr>
<tr>
<td>Moment 1, ( q_1 )</td>
<td>25% span</td>
<td>0.094</td>
</tr>
<tr>
<td>Moment 2, ( q_2 )</td>
<td>0.0</td>
<td>0.066</td>
</tr>
<tr>
<td>Shear</td>
<td>0.45</td>
<td>0.45</td>
</tr>
</tbody>
</table>

**Conditions**

For beams and slabs, 3 or more spans. (They may also be used for 2 span beams but support moment coefficients \( q_1 \), \( q_2 \), and internal shear coefficients \( 0.63 \) both taken.

Generally \( q_1 \), \( q_2 \), and the loading should be substantially uniformly distributed. Otherwise special arrangement of reinforcement is required.

Minimum span = 0.85 \( \times \) maximum (and design) span

Design moment of supports = \( \text{coeff} \times n \times \text{span} \)

Design shear at centreline of supports = \( \text{coeff} \times n \times \text{span} \) where \( n \) is a UDL with a single variable action

Design shear at ends of supports = \( \text{coeff} \times n \times \text{span} \) where \( n \) is characteristic permanent and variable actions in kN/m

\( q_1 \) and \( q_2 \) are dependent on use of BS EN 1990 Exp (6.10a) or Exp (6.10b). See Section 15.1.

For all full and alternate spans loaded cases use UK National Annex and 15% redistribution at supports.

**Key**

- All outer support 25% span relates to the UK Nationally Determined Parameter for BS EN 1992-1-1 9.2.1.2(1) for minimum percentage of span bending moment to be assumed at supports in beams in plane of connection. 15% can be applied in this case see BS EN 1992-1-1 9.2.1.2(1).
**Analysis Methods**

**Yield Line Method**

Equilibrium and work methods.

‘work method’

External energy expended by the displacement of loads

\[ \text{External energy} \]

Internal energy dissipated by the yield lines rotating

---

**Analysis Methods**

**Yield Line Method**

Suitable for:

- irregular layouts
- Slabs supported on 2 or 3 edges only

Detailed guidance and numerous worked examples contained in:

*Practical Yield Line Design*

Deflection design to simplified rules
### Analysis Methods

**Finite Element Method**

Suitable for:
- irregular layouts
- slabs with service openings
- post tensioned design (specialist software)

Common pitfalls:
- Use long term E-values (typically 1/3 to 1/2 short term value)
- Use cracked section properties (typically 1/2 gross properties) by adjusting E-value to suit
- Therefore appropriate E-values are usually 4 to 8 kN/mm²

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**Finite Element - Design moments**

![Finite Element Analysis Diagram]

- Middle strip
- Column strip
- Outer column strip
- Inner column strip

**Key**
- Section through bending moment diagram from FE output
- Design bending moment to BS 8110
- Averaging of bending moment
Design Constraints

Punching Shear - EC2: cl 6.4 and cl 9.4.3

- Traditional links

- Shear Rails

Deflection:

Wherever possible use the span/effective depth ratios, cl 7.4.2 (2)

Span is based on the longer span and the K factor is 1.2

Reduction factor for brittle finishes for spans greater than 8.5m
**Design Constraints**

**Moment Transfer from slab to column:**

Edge and corner columns have limited capacity to transfer moments from slab - redistribution may be necessary (Annex I.1.2 (5), EC2 cl 9.4.2 & TR 64)

\[
M_{t, \text{max}} = 0.17 b_e d^2 f_{ck}
\]

**Effective width, \( b_e \).**

---

**Flat slab worked example**

**Flexure**

From Worked Examples to EC2: Volume 1
Example 3.4.
**Introduction to worked example**

The slab is for an office where the specified load is 1.0 kN/m² for finishes and 4.0 kN/m² imposed (no partitions). Perimeter load is assumed to be 10 kN/m. Concrete is C30/37. The slab is 300 mm thick and columns are 400 mm square. The floor slabs are at 4.50 m vertical centres. A 2 hour fire rating is required.

This is example 3.4 of *Worked examples to Eurocode 2: Volume 1*.

**Design information**
- Nominal cover = 30mm

---

**Design strip along grid line C**

Determine the reinforcement - slab along grid line C.

Assume strip is 6 m wide
Slab is 300 mm deep
Flat slab

Worked example

For the previous flat slab example determine:

- Sagging reinforcement in the span 1-2

and

- Hogging reinforcement at support 2

---

**Analysis**

Actions:

\[ q_k = 0.30 \times 25 + 1.0 = 8.5 \text{ kN/m}^2 \]
\[ q_k = 4.0 \text{ kN/m}^2 \]
\[ n = 1.25 \times 8.5 + 1.5 \times 4.0 = 16.6 \text{ kN/m}^2 \]

Analysis: using coefficients from Concise Table 15.3:

(Adjacent spans are 9.6 and 8.6 m. 8.6/9.6 = 0.89: i.e. > 85% so using coefficients is appropriate.)

Effective span = 9.6 - 2 x 0.4/2 + 2 x 0.3/2 = 9.5 m

In panel: sagging moment,

\[ M_{Ed} = (1.25 \times 8.5 \times 0.09 + 1.5 \times 4 \times 0.100) \times 6.0 \times 9.5^2 = 842.7 \text{ kNm} \]

Along support 2: hogging moment

\[ M_{Ed} = 16.6 \times 0.106 \times 6.0 \times 9.5^2 = 952.8 \text{ kNm} \]

See Note to Concise Table 15.3 for support of 2-span slab
Division of moments

<table>
<thead>
<tr>
<th>Location</th>
<th>Negative moments</th>
<th>Positive moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column strip</td>
<td>60% – 80%</td>
<td>50% – 70%</td>
</tr>
<tr>
<td>Middle strip</td>
<td>40% – 20%</td>
<td>50% – 30%</td>
</tr>
</tbody>
</table>

Notes
The total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%
The distribution of design moments given in BS 8110 (column strip: hogging 75%, sagging 55%; middle strip: hogging 25%, sagging 45%) may be used

<table>
<thead>
<tr>
<th>Moment (kNm/m)</th>
<th>Column strip</th>
<th>Middle strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ve sagging</td>
<td>0.50 x 842.7/3.0 = 140.5 kNm/m</td>
<td>0.50 x 842.7/3.0 = 140.5 kNm/m</td>
</tr>
</tbody>
</table>

(50% taken in both column and middle strips)

From analysis

3.4.5 Design grid line C
Effective depth, d:
d = 300 – 30 – 20/2 = 260 mm

a) Flexure: column strip and middle strip, sagging

\[ M_{Ed} = 140.5 \, \text{kNm/m} \]
\[ K = \frac{M_{Ed}}{b_d^2 f_{ck}} = \frac{140.5 \times 10^6}{(1000 \times 260^2 \times 30)} = 0.069 \]
\[ z/d = 0.94 \quad (\text{Using Concise table 15.5}) \]
\[ z = 0.94 \times 260 = 244 \, \text{mm} \]
\[ A_e = \frac{M_{Ed}}{f_{ck}^2} \times z = \frac{140.5 \times 10^6}{(244 \times 500/1.15)} = 1324 \, \text{mm}^2/m \]
\[ (\rho = 0.51\%) \]

Try H20 @ 200 B1 (1570 mm²/m²)

\[ z = d \left[ 1 + (1 - 3.529K)^{0.5} \right]/2 = 260[1 + (1 - 3.529 \times 0.069)^{0.5}]/2 = 243 \, \text{mm} \]
Hogging Moments

<table>
<thead>
<tr>
<th></th>
<th>Column strip</th>
<th>Middle strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>-ve hogging</td>
<td>0.70 x 952.8/3.0 = 222.3 kNm/m</td>
<td>0.30 x 952.8/3.0 = 95.3 kNm/m</td>
</tr>
</tbody>
</table>

(70% taken in column strip and 30% in middle strip)

c) Flexure: column strip, hogging

\[ M_{Ed} = 222.3 \text{ kNm/m} \]

\[ K = \frac{M_{Ed}}{b'd'd_{ck}} = \frac{222.3 \times 10^6}{(1000 \times 260^2 \times 30)} = 0.109 \]

\[ z/d = 0.89 \quad \text{(Using Concise Table 15.5)} \]

\[ z = 0.89 \times 260 = 231 \text{ mm} \]

\[ A_o = M_{Ed}/f_{yd}z = \frac{222.3 \times 10^6}{(231 \times 500/1.15)} = 2213 \text{ mm}^2/\text{m} \]

\[ z = d \left[ 1 + (1 - 3.529K)^{0.5} \right]/2 = 260\left[ 1 + (1 - 3.529 \times 0.109)^{0.5} \right]/2 = 232 \text{ mm} \]

d) Flexure: middle strip, hogging

\[ M_{Ed} = 95.3 \text{ kNm/m} \]

\[ K = \frac{M_{Ed}}{b'd'd_{ck}} = \frac{95.3 \times 10^6}{(1000 \times 260^2 \times 30)} = 0.047 \]

\[ z/d = 0.95 \quad \text{(Using Concise Table 15.5)} \]

\[ z = 0.95 \times 260 = 247 \text{ mm} \]

\[ A_o = M_{Ed}/f_{yd}z = \frac{95.3 \times 10^6}{(247 \times 500/1.15)} = 887 \text{ mm}^2/\text{m} \]

\[ z = d \left[ 1 + (1 - 3.529K)^{0.5} \right]/2 = 260\left[ 1 + (1 - 3.529 \times 0.047)^{0.5} \right]/2 \]

\[ = 248 \text{ mm} \leq 0.95d \leq 247 \text{ mm} \]

\[ = 247 \]

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Reinforcement distribution

Total area of reinforcement:

\[ A_{s,tot} = 2213 \times 3 + 887 \times 3 = 9300 \text{ mm}^2 \]

50% \( A_{s,tot} = 9300/2 = 4650 \text{ mm}^2 \)

This is spread over a width of 1.5 m

\[ A_{s,req} = 4650/1.5 = 3100 \text{ mm}^2/m \]

Use H20 @ 100 ctrs T(3140 mm\(^2\)/m)

Remaining column strip:

\[ A_{s,req} = (2213 \times 3 - 4650)/1.5 = 1326 \text{ mm}^2/m \]

Use H20 @ 200 ctrs T(1570 mm\(^2\)/m)

Or use H16 @ 100 ctrs(1540 mm\(^2\)/m)

Middle strip: \( A_{s,req} = 887 \text{ mm}^2/m \)

Use H16 @ 200 ctrs T(1010 mm\(^2\)/m)

Or use H12 @ 100 ctrs (1130 mm\(^2\)/m)

Exercise

Lecture 5

Check an edge column for punching shear
Punching shear Exercise

Based on the flat slab in section 3.4 of Worked Examples to EC2: Volume 1

Punching shear at column C1

- 400 mm Square Column
- 300 mm flat slab C30/37 concrete

Design information:
- At C1 the ultimate column reaction is 609.5 kN
- Effective depths are 260mm & 240mm
- Reinforcement: $\rho_y = 0.0080$, $\rho_z = 0.0069$
**Punching shear exercise**

For the previous flat slab example:

a) Check the shear stress at the perimeter of column C1. The \( u_0 \) perimeter.

b) Check the shear stress at the basic perimeter, \( u_1 \).

c) Determine the distance of the \( u_{out} \) perimeter from the face of column C1.

d) Determine the area of shear reinforcement required on a perimeter, i.e. find \( A_{sw} \) for the \( u_1 \) perimeter.

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**Working space**
Working space

End of Lecture 5

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