# Practical Design to Eurocode 2

The webinar will start at 12.30

# Crack Control and Deflection

Lecture 6

28th October 2015

# Model Answers

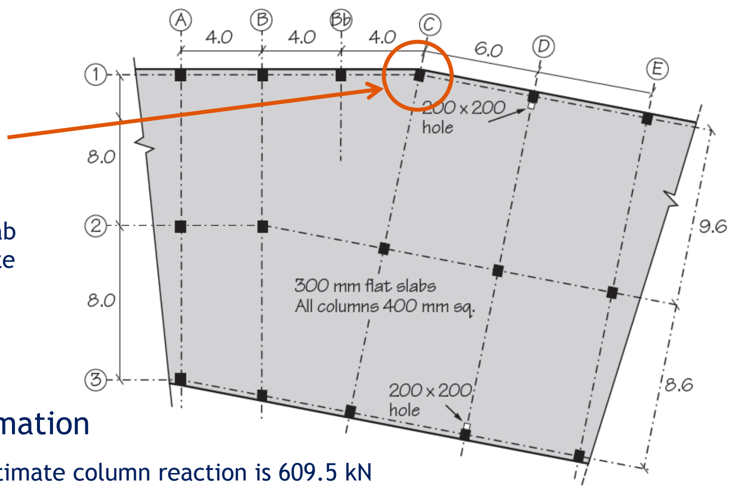
Lecture 5 Exercise:

Check an edge column for punching shear

## Punching shear at column C1

400 mm Square Column

300 mm flat slab
C30/37 concrete




### Design information

- At C1 the ultimate column reaction is 609.5 kN
- Effective depths are 260mm & 240mm
- Reinforcement: $\rho_{ly}$ = 0.0080, $\rho_{lz}$ = 0.0069

# Punching shear exercise

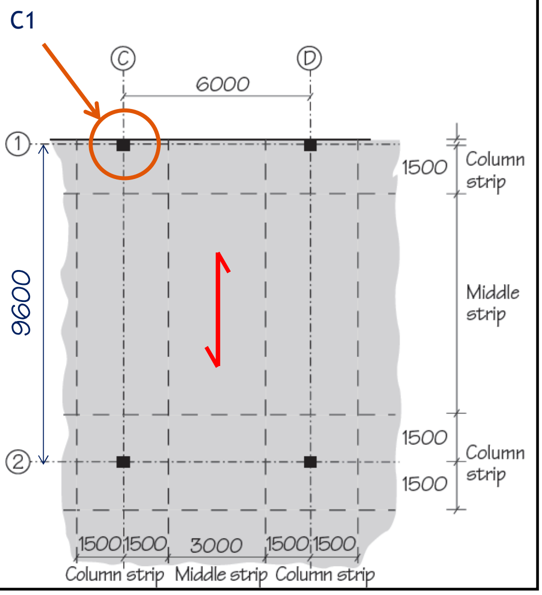
For the previous flat slab example:

a) Check the shear stress at the perimeter of column C1. The $u_0$ perimeter.

b) Check the shear stress at the basic perimeter, $u_1$.

c) Determine the distance of the $u_{out}$ perimeter from the face of column C1.

d) Determine the area of shear reinforcement required on a perimeter. i.e. find $A_{sw}$ for the $u_1$ perimeter.




# Solution

a) Check shear at the perimeter of the column

$v_{Ed} = \beta V_{Ed}/(u_0 d) < v_{Rd,max}$

$\beta = 1.40$

$d = (260 + 240)/2 = 250$ mm

$u_0 = c_2 + 3d < c_2 + 2c_1$ For edge columns

$u_0 = 400 + 3 \times 250 < 400 + 2 \times 400$

$u_0 = 1150$ mm

$v_{Ed} = 1.40 \times 609.5 \times 1000/(1150 \times 250)$

$= 2.97$ MPa

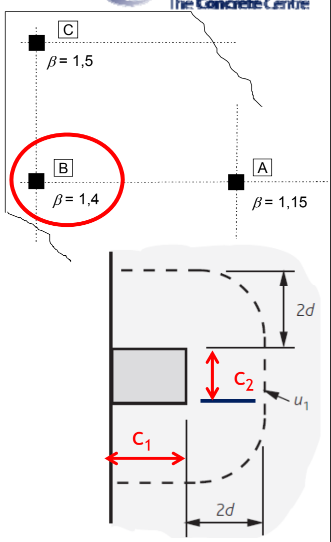
$v_{Rd,max} = 0.5\, \nu\, f_{cd}$

$= 0.5 \times 0.6(1-f_{ck}/250) \times \alpha_{cc}\, f_{ck}/\gamma_m$

$= 0.5 \times 0.6(1-30/250) \times 1.0 \times 30\,/1.5$

$= 5.28$ MPa

$v_{Ed} < v_{Rd,max}$ ...OK



## Solution
Rab's questions

We can calculate $\beta$ if we wish. See 6.4.3(4).
But that involves moments, $u_{1^*}$, $W_1$, $e_{par}$ etc.
$\beta$ = 1.4 is easy - if conservative.

a) Check shear at the perimeter of the column

$v_{Ed} = \beta V_{Ed}/(u_0 d) < v_{Rd,max}$

$\beta = 1.40$

$d = (260 + 240)/2 = 250$ mm

$u_0 = c_2 + 3d < c_2 + 2c_1$ For edge columns

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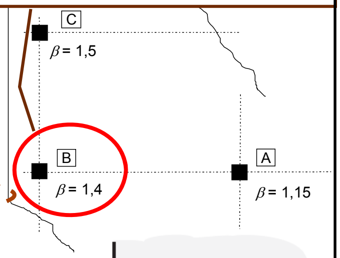
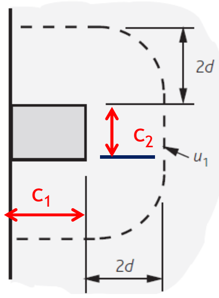
$= 0.5 \times 0.6(1 - 30/250) \times 1.0 \times 30/1.5$

$= 5.28$ MPa

$v_{Ed} < v_{Rd,max}$ ...OK




See Exp (6.53)

## Solution

b) Check shear at $u_1$, the basic control perimeter

$v_{Ed} = \beta V_{Ed}/(u_1 d) < v_{Rd,c}$

$\beta, V_{Ed}$ as before

$u_1 = c_2 + 2c_1 + \pi \times 2d$

$= 400 + 2 \times 400 + \pi \times 2 \times 250 = 2771$ mm

$v_{Ed} = 1.4 \times 609.5 \times 1000/(2771 \times 250) = 1.23$ MPa

$v_{Rd,c} = 0.12\, k(100\rho_l\, f_{ck})^{1/3}$

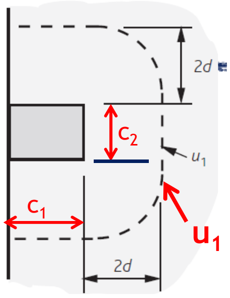
$k = 1 + (200/d)^{1/2} = 1 + (200/250)^{1/2} = 1.89$

$\rho_l = (\rho_{ly}\rho_{lx})^{1/2} = (0.0080 \times 0.0069)^{1/2} = 0.0074$

$v_{Rd,c} = 0.12 \times 1.89(100 \times 0.0074 \times 30)^{1/3} = 0.64$ MPa

$v_{Ed} > v_{Rd,c}$ ?

1.23 MPa > 0.64 MPa ... Therefore punching shear reinf. required




ρ from flexural calcs

NA check:

$v_{Ed} \leq 2.0 v_{Rd,c}$ at basic control perimeter

1.23 MPa ≤ 2 x 0.64 MPa = 1.28 MPa - OK

## Solution

c) Perimeter at which punching shear no longer required

$u_{out} = \beta V_{Ed}/(dv_{Rd,c})$

$= 1.4 \times 609.5 \times 1000/(250 \times 0.64)$

$= 5333$ mm

Rearrange: $u_{out} = c_2 + 2c_1 + \pi r_{out}$

$r_{out} = (u_{out} - (c_2 + 2c_1))/\pi$

$= (5333 - 1200)/\pi = 1315$ mm

Position of outer perimeter of reinforcement from column face:

$r = 1315 - 1.5 \times 250 = 940$ mm

Maximum radial spacing of reinforcement:

$s_{r,max} = 0.75 \times 250 = 187$ mm, say 175 mm

## Solution

d) Area of reinforcement

$A_{sw} \geq (v_{Ed} - 0.75v_{Rd,c})s_r u_1/(1.5f_{ywd,ef})$

$f_{ywd,ef} = (250 + 0.25d) = 312$ MPa

$A_{sw} \geq (1.23 - 0.75 \times 0.64) \times 175 \times 2771/(1.5 \times 312)$

$\geq$ 777 mm² per perimeter

Within the $u_1$ perimeter the link spacing around a perimeter,

$s_t \leq 1.5d = 1.5 \times 250 = 375$mm

Outside the $u_1$ perimeter the link spacing around a perimeter,

$s_t \leq 2d = 500$ mm

Use say $s_{t,max} = 350$ mm

Minimum area of a link leg:

$A_{sw,min} \geq (0.053\, s_r\, s_t \sqrt{(f_{ck})})/f_{yk} = 0.053 \times 175 \times 350 \times \sqrt{30} / 500$

$\geq$ 36 mm²

Use either H8s (50 mm²) and this would need 16 per perimeter
or H10s (78.5 mm²) and 10 per perimeter.
@ 350 mm tangential spacing and @175 mm radial spacing

# Crack Control and Deflection

Lecture 6

28th October 2015

## Outline - Lecture 6

- General
- Crack control
- Deflection
- Worked Example
- Design Exercise

# General

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## What does Eurocode 2 Cover?

Cl. 7.1

Concise 10.1

- General (7.1)

7.1(1) Generally SLS limit states are:

- stress limitation (see 7.2),
- control of cracking (see 7.3), and
- control of deflections (see 7.4)

Vibration may be important but is not covered.

## What does Eurocode 2 Cover?

Cl. 7.1

Concise 10.1

7.1(2) In the calculation of stresses and deflections, cross-sections should be **assumed to be uncracked** provided that the flexural tensile stress does not exceed $f_{ct,eff}$.

The value of $f_{ct,eff}$ may be taken as $f_{ctm}$ or $f_{ctm,fl}$ provided that the calculation for minimum tension reinforcement is also based on the same value.

For the purposes of calculating crack widths and tension stiffening $f_{ctm}$ should be used.

[From Table 3.1, $f_{ctm} = 0.30 f_{ck}^{(2/3)}$ for ≤ C50/60]

## What does Eurocode 2 Cover?

7.2 Stress limitation

Unacceptable cracking may be assumed to be avoided if $\sigma_{ck} < 0.6 f_{ck}$ and $\sigma_{sk} < 0.8 f_{yk}$

BUT:- PD 6687 Cl. 2.20: "Stress checks in reinforced concrete members have not been required in the UK for the past 50 years or so and there has been no known adverse effect. Provided that the design has been carried out properly for ultimate limit state there will be no significant effect at serviceability in respect of longitudinal cracking"

7.3 Control of cracking

7.4 Control of deflections (7.4)

# Crack control

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## Control of Cracking
Cl. 7.3

In Eurocode 2 cracking is controlled in the following ways:

- Minimum areas of reinforcement Cl 7.3.2 & Exp (7.1)
- Limiting crack widths.
  $w_{kmax}$ is determined from Table 7.1N (in the UK from Table NA.4)
  These limits can be met by either:
  - 'deemed to satisfy' rules (Cl. 7.3.3)
  - direct calculation (Cl. 7.3.4) – design crack width is $w_k$

Note: slabs ≤ 200mm depth are OK if $A_{s,min}$ is provided.

## Control of Cracking

Cl. 7.3.2 10.2

Minimum areas of reinforcement

$$A_{s,min}\sigma_s = k_c k f_{ct,eff} A_{ct}$$ Exp (7.1)

where

$\sigma_s$ = Stress in the reinforcement ( = $f_{yk}$)

$k_c$ = Stress distribution factor:
= 1.0 for pure tension
= $0.4(1-\sigma_c / (k_1(h/h^*)f_{ct,eff})$
where:
$\sigma_c$ = $N_{Ed}/bh$
$k_1$ = 1.5 for compression
= $0.66h^*/h$ for tension
$h^*$ = Max(h, 1000 mm)
$f_{ct,eff} = f_{ctm} = 0.30f_{ck}^{(2/3)}$ for ≤ C50/60 (or lower if cracking is expected before 28 days)

k = Coefficient to allow for 'non-uniform equilibrating stresses'
= 1.0 for h = 300 mm
= 0.65 for h = 800 mm. Interpolate between.

$A_{ct}$ = area of concrete within tensile zone.
The tensile zone is that part of the section which is calculated to be in tension just before formation of the first crack

Essentially: force in un-yeilded reinforcement ≥ tensile force in concrete just before it cracks.

Exp (7.1) is the same as $A_{smin} \geq 0.26f_{ctm}b_t d/f_{yk}$ used in $A_{smin}$ for beams in 9.2.1.1(1). See later – Detailing.

## Control of Cracking

Table 7.1 but - use Table NA.4

### Crack Width Limits

| Recommended values of $w_{max}$ | | |
|---|---|---|
| Exposure class | RC or unbonded PSC members | Prestressed members with bonded tendons |
| | Quasi-permanent load | Frequent load |
| X0,XC1 | 0.3* | 0.2 |
| XC2,XC3,XC4 | 0.3 | |
| XD1,XD2,XS1, XS2,XS3 | | Decompression |
| * Does not affect durability, may be relaxed where appearance is not critical (eg use 0.4 mm) | | |

# Crack Control Without Direct Calculation
Cl 7.3.3

Crack control (due to flexure) may be achieved in two ways:

- limiting the maximum bar diameter using Table 7.2
- limiting the maximum bar spacing using Table 7.3

| Steel stress ($\sigma_s$) MPa | $w_{max}$ = 0.4 mm | | | $w_{ma}$ = 0.3 mm | | |
|---|---|---|---|---|---|---|
| | Maximum bar size (mm) | | Maximum bar spacing (mm) | Maximum bar size (mm) | | Maximum bar spacing (mm) |
| 160 | 40 | | 300 | 32 | | 300 |
| 200 | 32 | OR | 300 | 25 | OR | 250 |
| 240 | 20 | | 250 | 16 | | 200 |
| 280 | 16 | | 200 | 12 | | 150 |
| 320 | 12 | | 150 | 10 | | 100 |
| 360 | 10 | | 100 | 8 | | 50 |

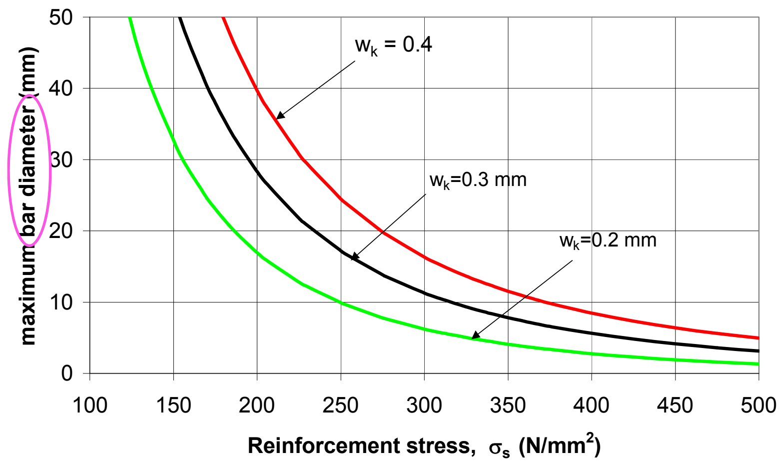
NB: Where cracking is due to restraint use only limiting max. bar size

# Maximum Bar Diameters

Cl. 7.3.3

Table 10.1

Crack Control



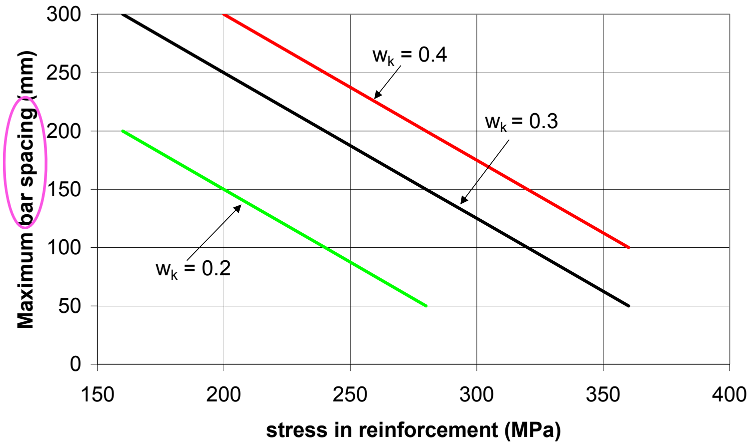

(Stress due to quasi-permanent actions)

## Maximum Bar Spacings

Cl. 7.3.3

Table 10.2

### Crack Control




(Stress due to quasi-permanent actions)

## Calculation of crack widths

Cl 7.3.4

Crack width, $w_k = s_{r,max}\, \varepsilon_{cr}$ Exp (7.8)

where

$s_{r,max}$ = Maximum crack spacing = $3.4c + 0.425\,(k_1 k_2 \phi / \rho_{p,eff})$ Exp (7.11)

where

c = nominal cover, $c_{nom}$
$k_1$ = 0.8
$k_2$ = 1.0 for tension (e.g. from restraint)
= 0.5 for bending
= $(\varepsilon_1 + \varepsilon_2)/2\varepsilon_1$ for combinations of bending and tension
$\phi$ = diameter of the bar in mm.
$\rho_{p,eff}$ = $A_s/A_{c,eff}$
$A_{c,eff}$ for each face is based on {0.5h; 2.5(c + 0.5$\phi$); (h - x)/3} where h = thickness of section and x = depth to neutral axis.

$\varepsilon_{cr}$ = Crack-inducing strain
= Strain between cracks
= Mean strain in steel – mean strain in concrete, $(\varepsilon_{cs} - \varepsilon_{cm})$. . . . . .

## Calculation of crack widths: $\varepsilon_{cr}$

$\varepsilon_{cr} = (\varepsilon_{cs} - \varepsilon_{cm})$:

For **flexural** (and applied tension) crack-inducing strain

$$\varepsilon_{cr} = [\sigma_s - k_t\,(f_{ct,eff}/\rho_{p,eff})\,(1 + \alpha_e\,\rho_{p,eff}]\,/E_s \geq 0.6\,(\sigma_s)/E_s \quad \text{Exp (7.9)}$$

where

$\sigma_s$ = SLS stress in the reinforcement

$k_t$ = Factor for duration of load
= 0.6 for short-term loading
= 0.4 for long-term loading

$f_{ct,eff}$ = $f_{ctm} = 0.30 f_{ck}^{(2/3)}$ for ≤ C50/60 Table 3.1
(or lower if cracking is expected before 28 days)

$\rho_{p,eff}$ = $A_s/A_{c,eff}$
$A_{c,eff}$ for each face is based on Min[0.5h; 2.5(c + 0.5ϕ); (h - x)/3]
where
h = thickness of section and
x = depth to neutral axis.

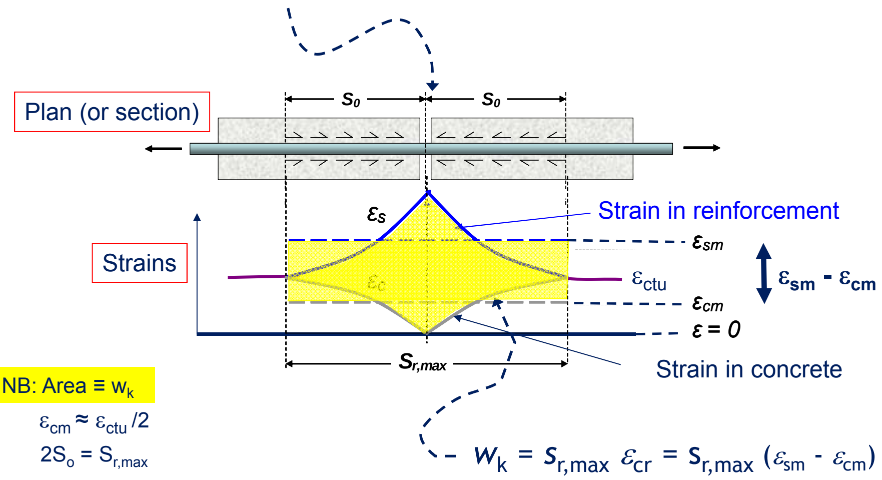
$\alpha_e$ = Modular ratio $E_s/E_{cm}$
Where $E_{cm} = 22(f_{cm}/10)^{0.3}$ Table 3.1
Typical values of $\alpha_e$ are 6 @ 3 days, 7 at 28 days and 12 long-term. When cracking occurs no creep has taken place so $\alpha_e$ = 7 is recommended in crack width calculations

$E_s$ = 200,000 MPa

## Calculation of crack widths: $\varepsilon_{cr}$

$\varepsilon_{cr} = (\varepsilon_{cs} - \varepsilon_{cm})$:

Consider a crack in a section:



## Restraint cracking

A section will crack if:

$$\varepsilon_r = R_{ax}\,\varepsilon_{free} = K[([\alpha_c T_1 + \varepsilon_{ca})]\,R_1 + ([\alpha_c T_2\,R_2)] + \varepsilon_{cd}\,R_3] > \varepsilon_{ctu}$$

where

CIRIA C660 Cl 3.2

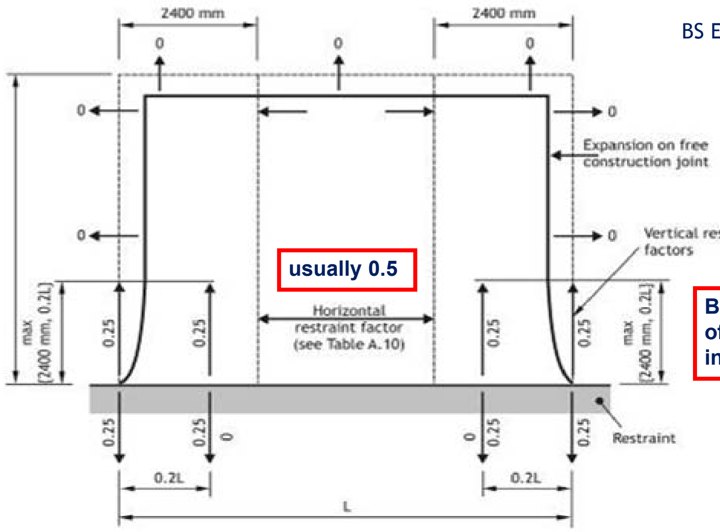
| | | |
|---|---|---|
| K | = | allowance for creep |
| | = | 0.65 when R is calculated using CIRIA C660 |
| | = | 1.0 when R is calculated using BS EN 1992-3 |
| $\alpha_c$ | = | coefficient of thermal expansion (See CIRIA C660 for values). See Table A6 for typical values |
| $T_1$ | = | difference between the peak temperature of concrete during hydration and ambient temperature °C (See CIRIA C660). Typical values are noted in Table A7 |
| $\varepsilon_{ca}$ | = | Autogenous shrinkage strain - value for early age (3 days: see Table A9) |
| $R_1$, $R_2$, $R_3$ | = | restraint factors. See Section A5.6<br>For edge restraint from Figure L1 of BS EN 1992-3 for short- and long-term thermal and long-term drying situations. For base-wall restraint they may be calculated in accordance with CIRIA C660. Figure L1 may be used with CIRIA C660 methods providing an adjustment for creep is made (See Figure A2 and note).<br>For end restraint, where the restraint is truly rigid 1.0 is most often used, for instance in infill bays. This figure might be overly pessimistic for piled slabs. |
| $T_2$ | = | long-term drop in temperature after concreting, °C. $T_2$ depends on the ambient temperature during concreting. The recommended values from CIRIA C660 for $T_2$ are 20°C for concrete cast in the summer and 10°C for concrete cast in winter. These figures are based on HA BD 28/87[60] based on monthly air temperatures for exposed bridges. Basements are likely to follow soil temperatures so $T_2$ = 12°C may be considered appropriate at depth. |
| $\varepsilon_{cd}$ | = | drying shrinkage strain, dependent on ambient RH, cement content and member size (see BS EN 1992-1-1 Exp. (3.9) or CIRIA C660 or Table A10). CIRIA C660 alludes to 45% RH for internal conditions and 85% for external conditions. |
| $\varepsilon_{ctu}$ | = | tensile strain capacity may be obtained from Eurocode 2 or CIRIA C660 for both short term and long term values |

See TCC's *Concrete Basements* for details

## Restraint cracking

### Restraint factors

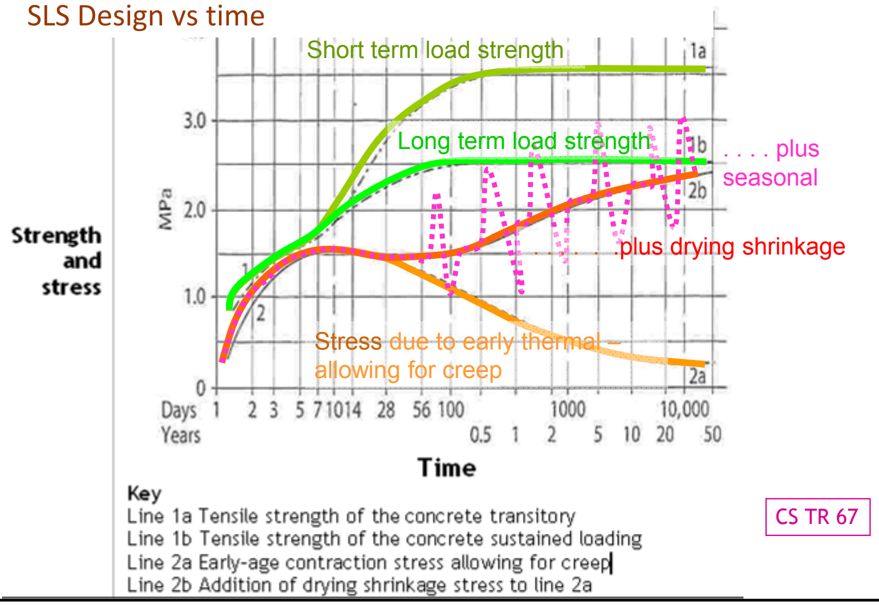
BS EN 1992-3 Annex L




**Beware: effects of creep included**

# Restraint cracking

SLS Design vs time




CS TR 67

# Restraint cracking

Test for restraint cracking

A section will crack if:

$$\varepsilon_r = R_{ax}\,\varepsilon_{free} = K[([\alpha_c T_1 + \varepsilon_{ca})]\,R_1 + ([\alpha_c T_2\,R_2)] + \varepsilon_{cd}\,R_3] > \varepsilon_{ctu}$$

CIRIA C660 Cl 3.2

Short term (≡ 3 days)

Medium term (≡ 28 days)

long term (≡ > 10000 days)

where $K$ = allow... creep
= ...d using CIRIA C660
... using BS EN 199...3

$\alpha_c$ ... xpansion (See CIRIA C660 for va...
$T_1$ = ... the ... of co...
temperature C (See ... e n...

$\varepsilon_{ca}$ = Autogenous shrink... days: see Table A7)

$R_1$, $R_2$, $R_3$ = restraint factors. S...
For edge restraint fr... -3 for short- and lon... term thermal and long-term drying situations. For base-wall restraint they ma... d in accordance with CIRIA C660. Figure L1 may be used with CIR... ng an adjustment for creep is made (See Figure A2 an...
For end restraint ... used, for instance in infill bays. This ...

$T_2$ = long-term drop ... s on the ambient temperature dur... from CIRIA C660 for $T_2$ are 20° C for concrete cast ... for concrete cast in winter. These figures are based on HA BD 2... monthly air temperatures for exposed bridges. Basements are likely to follow ... temperatures so $T_2$ = 12° C may be considered appropriate at depth.

$\varepsilon_{cd}$ = drying shrinkage strain, dependent on ambient RH, cement content and member size (see BS EN 1992-1-1 Exp. (3.9) or CIRIA C660 or Table A10). CIRIA C660 alludes to 45% RH for internal conditions and 85% for external conditions.

$\varepsilon_{ctu}$ = tensile strain capacity may be obtained from Eurocode 2 or CIRIA C660 for both short term and long term values

**Assume it cracks!!**

There are alternatives but ..........

Use min steel which can be approx. 0.58%

# Restraint cracking

$\varepsilon_{cr} = (\varepsilon_{cs} - \varepsilon_{cm})$:

Restraint: Water retaining structures etc.

Early age crack-inducing strain

$$\varepsilon_{cr} = K[\alpha_c T_1 + \varepsilon_{ca}]\, R_1 - 0.5\, \varepsilon_{ctu}$$ CIRIA C660 Cl 3.2

Long term crack-inducing strain

$$\varepsilon_{cr} = K[(\alpha_c T_1 + \varepsilon_{ca}) R_1 + \alpha_c T_2 R_2 + \varepsilon_{cd} R_3] - 0.5\, \varepsilon_{ctu}$$ CIRIA C660 Cl 3.2

End restraint crack-inducing strain

$$\varepsilon_{cr} = 0.5 \alpha_e\, k_c k f_{ct,eff}\, [1 + (1/\alpha_e\, \rho)]\, / E_s$$ EN 1992-3 Exp (M.1)

# Calculation of crack widths: $\varepsilon_{cr}$

## Summary:

Crack width, $w_k = s_{r,max}\, \varepsilon_{cr}$ Exp (7.8)

where

$s_{r,max}$ = Maximum crack spacing = $3.4c + 0.425\, (k_1 k_2 \phi / \rho_{p,eff})$ Exp (7.11)

where

| | | |
|---|---|---|
| c | = | nominal cover, $c_{nom}$ |
| $k_1$ | = | 0.8 (CIRIA C660 suggests 1.14) |
| $k_2$ | = | 1.0 for tension (e.g. from restraint) |
| | = | 0.5 for bending |
| | = | $(\varepsilon_1 + \varepsilon_2)/2\varepsilon_1$ for combinations of bending and tension |
| $\phi$ | = | diameter of the bar in mm. |
| $\rho_{p,eff}$ | = | $A_s/A_{c,eff}$<br>$A_{c,eff}$ for each face is based on {0.5h; 2.5(c + 0.5$\phi$); (h - x)/3} where h = thickness of section and x = depth to neutral axis. |

$\varepsilon_{cr}$ = Crack-inducing strain

= Strain between cracks

= Mean strain in steel - mean strain in concrete, $(\varepsilon_{cs} - \varepsilon_{cm})$. . . . . .

. . . . . . flexural or restraint

# Deflection

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## Deflection Limits

Cl. 7.4.1

According to EN 1990, Deflection Limits should be agreed with the Client. But in EN 1992-1-1, Cl 7.4.1 the following limits "should generally result in satisfactory performance of buildings …" (based on ISO 4356).

In EC2 the deflection limits implied are:

- Span/250 under quasi-permanent loads to avoid impairment of appearance and general utility
- Span/500 *after construction* under the quasi-permanent loads to avoid damage to adjacent parts of the structure.

## Deflection Control

Cl. 7.4.2 , 7.4.3 10.5

**Deflection control may be achieved by the following methods:**

- Using 'simplified' span-to-effective depth limits from the code (control of deflection without calculation)
- Calculation

## Basic span/effective depth ratios

Cl 7.4.2 & Exp (7.16a & b) 10.5.2

Basic l/d ratios:

$$\frac{l}{d} = K\left[11 + 1{,}5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3{,}2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right] \quad \text{if } \rho \leq \rho_0$$

$$\frac{l}{d} = K\left[11 + 1{,}5\sqrt{f_{ck}}\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}\right] \quad \text{if } \rho > \rho_0$$

$K$ factor taking account of the different structural systems

$\rho_0$ reference reinforcement ratio = $\sqrt{f_{ck}}\ 10^{-3}$

$\rho$ required tension reinforcement ratio at mid-span to resist the moment due to the design loads (at support for cantilevers)

$\rho'$ required compression reinforcement ratio at mid-span to resist the moment due to design loads (at support for cantilevers)

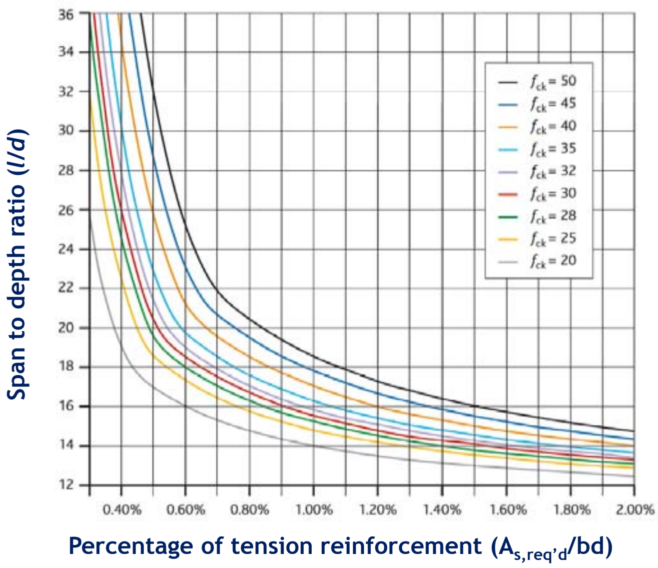
## Basic span/effective depth ratios

Table 7.4(N) use Table NA.5 Table 10.3

| Structural system | K | ρ = 1.5% | ρ = 0.5% |
|---|---|---|---|
| S.S. beam or slab | 1.0 | 14 | 20 |
| End span | 1.3 | 18 | 26 |
| Interior span | 1.5 | 20 | 30 |
| Flat slab | 1.2 | 17 | 24 |
| Cantilever | 0.4 | 6 | 8 |

## Basic span/effective depth ratios

EC2: Table 7.4(N) use Table NA.5




This graph has been produced for K = 1.0

K factors:

| Structural System | K |
|---|---|
| Simply supported | 1.0 |
| End span | 1.3 |
| Interior Span | 1.5 |
| Flat Slab | 1.2 |

Concise Table 10.3 & Fig 15.2, How to Beams: Fig 7

# Adjustments to *K*

EC2: cl 7.4.2 & NA

Concise 10.5.2

- **F1 - Flanged sections**

  where the ratio of the flange breadth to the rib breadth exceeds 3, the values of l/d given by Expression (7.16) should be multiplied by 0.8. {For $b_{eff}/b_w$ from 1 to 3 interpolate}

- **F2 - Long spans with brittle partitions**
  - For slabs (other than flat slabs), with spans exceeding 7.0 m, which support partitions liable to be damaged by excessive deflections, the values of l/d given by Expression (7.16) should be multiplied by 7.0/ $l_{eff}$ ($l_{eff}$ in metres, see 5.3.2.2 (1)).
  - For flat slabs, with spans exceeding 8.5 m, which support partitions liable to be damaged by excessive deflections, the values of l/d given by Expression (7.16) should be multiplied by 8.5 / $l_{eff}$ ($l_{eff}$ in metres, see 5.3.2.2 (1)).

- **F3 - $\sigma_s$ Steel stress under service load**

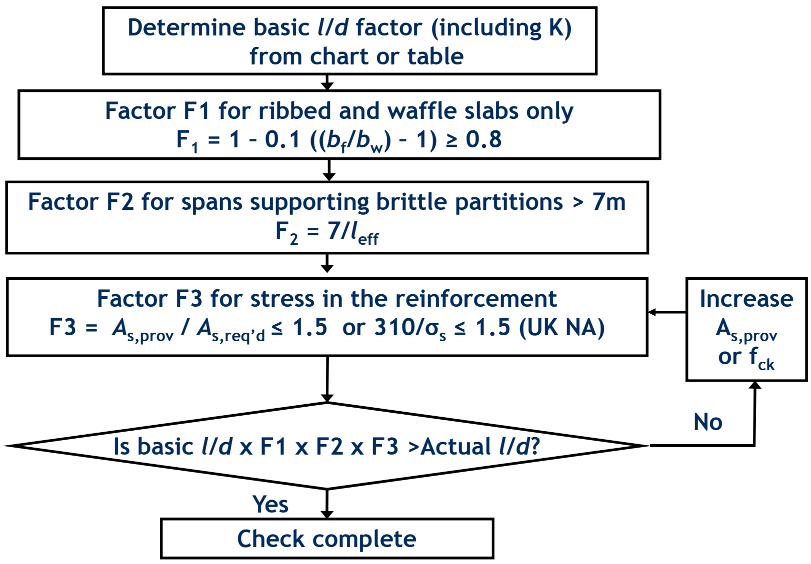
  May be adjusted by $310/\sigma_s \leq 1.5$ or $A_{s,prov}/A_{s,req} \leq 1.5$

  where $\sigma_s$ calculated using characteristic loads.

These are in cl 7.4.2 (2). F1, F2 and F3 is just TCC nomenclature

# Span/effective depth ratios

## Flow Chart



## Calculating deflection

Cl 7.4.3 & Exp (7.18)

EN 1992-1-1 (and MC2010) states that an adequate prediction of behaviour in a discreet element is given by:

$$\alpha = \zeta\alpha_{II} + (1-\zeta)\alpha_{I} \qquad \text{Exp (7.18)}$$

where:

$\alpha$ = the phenomenon, e.g. curvature

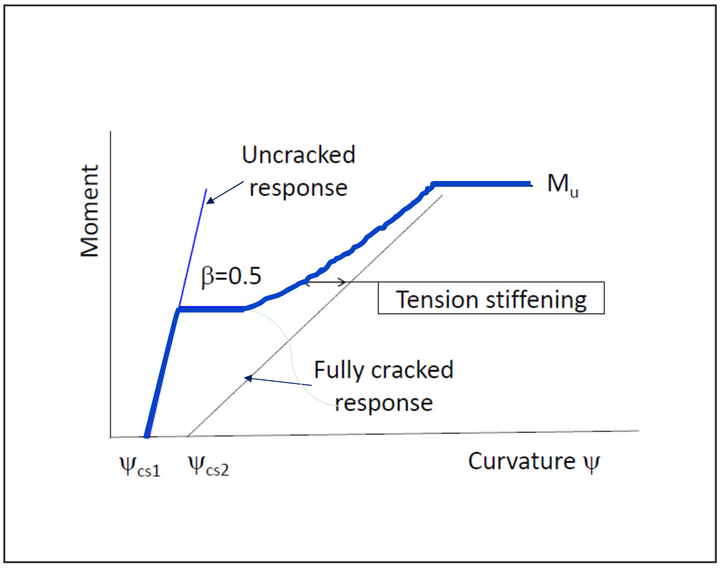
$\alpha_{II}$ = cracked phenomenon

$\alpha_{I}$ = uncracked phenomenon

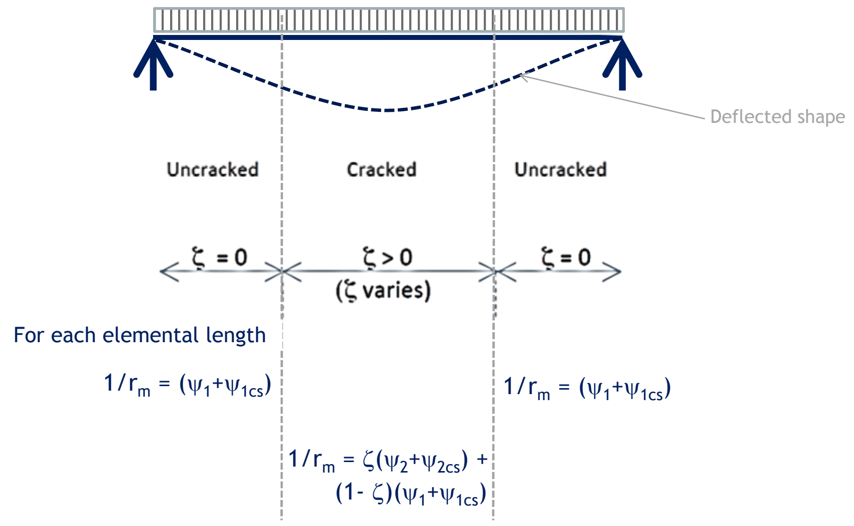
$\zeta$ = the distribution factor

i.e. somewhere between fully uncracked and fully cracked

## Consider moment: curvature



## Consider a s.s. beam under a udl




## Calculating deflection

Cl 7.4.3 & Exp (7.18)

So for deflection, an adequate prediction of behaviour and the mean curvature in a discreet element is given by:

$$1/r_m = \zeta(\psi_2+\psi_{2cs}) + (1-\zeta)(\psi_1+\psi_{1cs})$$

where

$r_m$ = mean radius

$\zeta = 1-\beta(M_{cr}/M)^2$

where

$\beta$ = 1.0 for short –term and 0.5 for long-term loading.

For construction loads, conservatively $\beta = 0.70$

$M_{cr}$ = cracking moment

M = Moment

where . . . . . .

## Calculating deflection

Cl 7.4.3 & Exp (7.18)

$$1/r_m = \zeta(\psi_2+\psi_{2cs}) + (1-\zeta)(\psi_1+\psi_{1cs})$$

where

$\psi_1 = M/E_{ceff}I_1$ = curvature of uncracked section

$\psi_2 = M/E_{ceff}I_2$ = curvature of cracked section

$E_{ceff} = E_{cm}/(1+\phi)$

where

$E_{cm}$ = modulus at 28 days

$\phi$ = creep coefficient

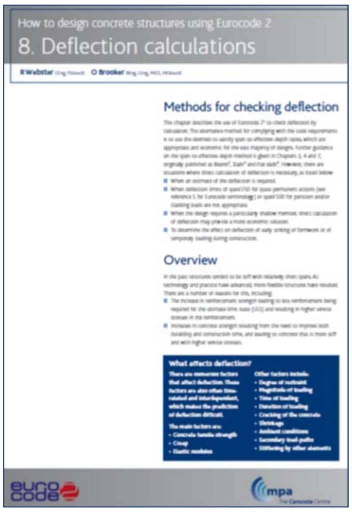
$I_1$, $I_2$ = inertias

$\psi_{1cs}$, $\psi_{2cs}$ = shrinkage curvature

## Calculating deflection

This 'rigorous' method is described in greater detail elsewhere [Concrete Society. *Deflections in concrete slabs and beams*, TCC *How to design concrete structures using Eurocode 2, #8. Deflection calculations*]. It is backed by site based research [Vollum].

The method involves numerical integration, which is tedious by hand but can, of course, be undertaken by computer software, notably by spreadsheets.


How to design concrete structures using Eurocode 2

8. Deflection calculations

Methods for checking deflection

Overview

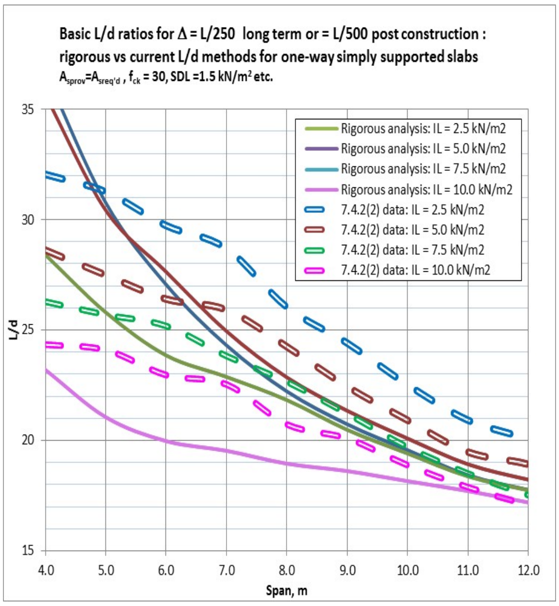
What affects deflection?

## L/d vs calculating deflection

Making lots of reasonable assumptions a comparison can be drawn.

As may been seen for solid slabs the agreement between the current L/d method (to Cl 7.4.2(2)) and the current *rigorous* method is:

- reasonable for high imposed loads and long spans but
- not too good at low spans or for low imposed loads.
- L/d is generally conservative - except for low loads and long spans!(say > 6.0 m simply supported span or > 8 m continuous span)




## L/d vs calculating deflection

Given:

- the complexity and the variability of the concrete as a material,
- loading and environment,
- assumptions made in design,

it is perhaps unsurprising that there is a difference.

However, as stated in EN1992-1-1, it appears that the use of L/d methods ‘will be adequate for avoiding deflection problems in normal circumstances’.

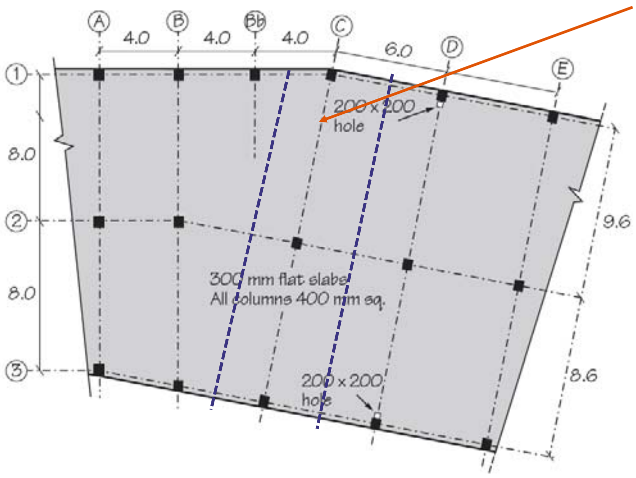
The ‘rigorous’ method is necessary in unusual circumstances or where deflection limits other than those implicit in the simplified methods are appropriate.

# Worked example

Lecture 6

## Worked example : problem

For the slab strip along grid line C, check deflection is within design limits and ensure the crack widths in the bottom of this office slab are also limited.




Check slab along grid line C for deflection and cracking

Assume:

- strip is 6 m wide
- $A_{s,req}$ = 1310 mm² B
- d = 300 - 30 - 20/2 = 260 mm
- $\gamma_G$ = 1.25

# Workshop problems
## Design information

### 3.4.1 Actions

| | | kN/m² |
|---|---|---|
| Permanent: | | |
| Self-weight 0.30 x 25 | | = 7.5 |
| Finishes | | = 1.0 |
| | Total | $g_k$ = 8.5 |
| Variable: | | |
| Offices | | $q_k$ = 4.0[‡] |

### 3.4.4 Analysis grid line C

Effective spans:

9600 − 2 x 400/2 + 2 x 300/2 = 9500 mm

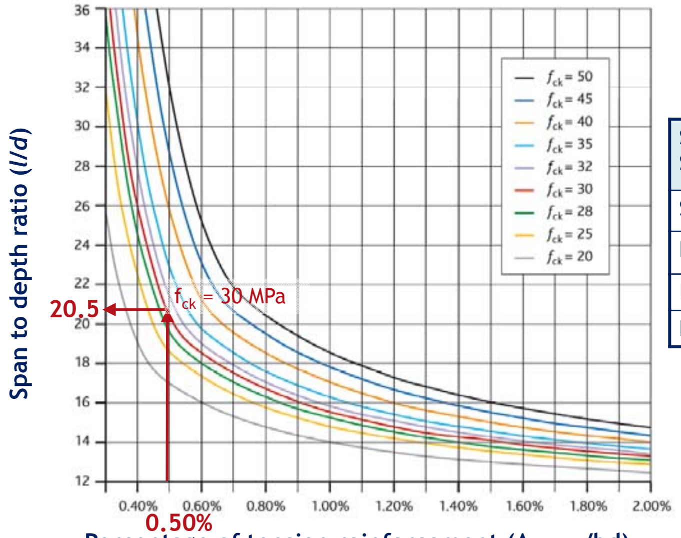
8600 − 2 x 400/2 + 2 x 300/2 = 8500 mm

# Worked example :
## L/d check

Check: basic $l/d$ x F1 x F2 x F3 ≥ actual $l/d$

1. Determine basic $l/d$
   The reinforcement ratio, $\rho = A_{s,req}/bd$
   = 1310 x 100/(1000 x 260)
   = 0.50%

# Basic Span-to-Depth Ratios
## (for simply supported condition)




How To 3: Figure 5

This graph has been produced for K = 1.0

| Structural System | K |
|---|---|
| Simply supported | 1.0 |
| End span | 1.3 |
| Interior Span | 1.5 |
| Flat Slab | 1.2 |

# Worked example :
## L/d check

Check: basic $l/d$ x F1 x F2 x F3 ≥ actual $l/d$

1. **Determine basic $l/d$**

   The reinforcement ratio, $\rho = A_{s,req}/bd$ = 1310 x 100/(1000 x 260)

   = 0.50%

   From graph, basic $l/d$ = 20.5 x 1.2 = 24.6 (*K* = 1.2 for flat slab)

2. **Determine Factor F1**

   F1 = 1.0

   **For flanged sections where the ratio of the flange breadth to the rib breadth exceeds 3, the values of *l/d* given by Expression (7.16) should be multiplied by 0.8.**

3. **Determine Factor F2**

   (Assuming no brittle partitions)

   F2 = 1.0

   **For flat slabs, with spans exceeding 8.5 m, which support partitions liable to be damaged by excessive deflections, the values of L/d given by Expression (7.16) should be multiplied by 8.5 / $l_{eff}$ ($l_{eff}$ in metres, see 5.3.2.2 (1)). Here there are no brittle finishes.**

# Worked example :

## L/d check

**4. Determine Factor F3:**

**Steel stress under service load: use $A_{s,prov}/A_{s,req} \leq 1.5$**

$A_{s,req}$ = 1310 mm$^2$ (ULS)

By inspection $A_{s,prov}$ should be > $A_{s,req}$ Try H16 @ 100 c/c (2010 mm$^2$) to control deflection:

F3 = $A_{s,prov}$ / $A_{s,req}$ = 2010 / 1310 = 1.53 ≤ 1.5

**5. Check L/d**

Basic $l/d$ x F1 x F2 x F3 ≥ actual $l/d$

24.6 x 1.0 x 1.0 x 1.5 ≥ 9500 / 260

36.9 ≥ 36.5 Therefore (just) OK

# Worked example :

## Crack Control Without Direct Calculation

EC2: Cl. 7.3.3

For the Worked example, check cracking in bottom of the flat slab.

$g_k$ = 8.5 kN/m$^2$ $q_k$ = 4.0 kN/m$^2$ $g_k/q_k$ = 2.1

$\Psi_2$ = 0.3 (office loading (remember?) $\gamma_G$ = 1.25

$\delta$ = 1.0

$A_{s,req}$ = 1310 mm$^2$/m

$A_{s,prov}$ = H16 @ 100 = 2010 mm$^2$/m

In order to use Tables 7.2N or 7.3N we need to determine the service stress in the bars either by:

- calculation - e.g. using

$$\frac{\text{Service stress}}{\text{Ultimate stress}} = \frac{G_k + \psi_2 Q_{k,1}}{\gamma_G G_k + \gamma_Q Q_{k,1}} \cdot \frac{1}{\delta}$$

or

- using a graph for the relevant $g_k/q_k$, $\Psi_2$ and $\gamma_G$ :

# Worked example :

## Crack Control Without Direct Calculation

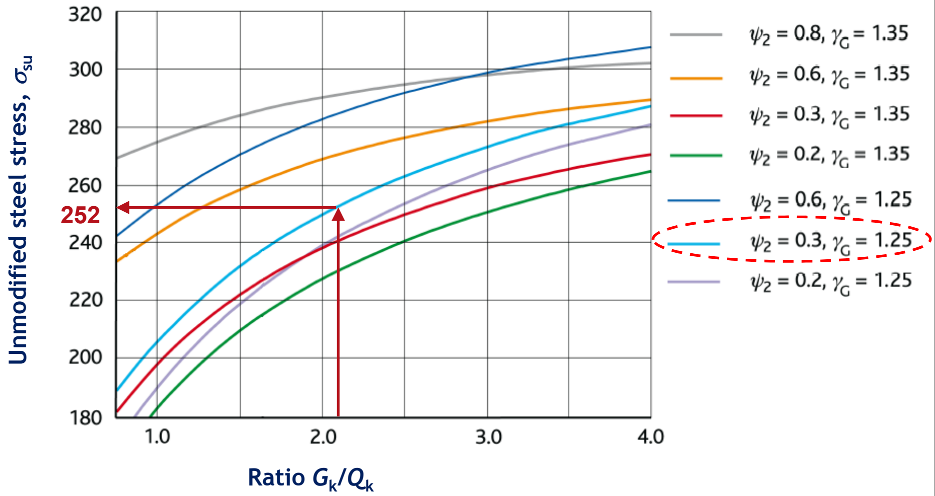
ψ factors (remember?)

| Action | $\psi_0$ | $\psi_1$ | $\psi_2$ |
|---|---|---|---|
| Imposed loads in buildings, | | | |
| Category A : domestic, residential | 0.7 | 0.5 | 0.3 |
| **Category B : office areas** | 0.7 | 0.5 | **0.3** |
| Category C : congregation areas | 0.7 | 0.7 | 0.6 |
| Category D : shopping areas | 0.7 | 0.7 | 0.6 |
| Category E : storage areas | 1.0 | 0.9 | 0.8 |
| Category F : traffic area, ≤ 30 kN | 0.7 | 0.7 | 0.6 |
| Category G : traffic area, 30-160 kN | 0.7 | 0.5 | 0.3 |
| Category H : roofs | | 0 | 0 |
| Snow load: H ≤ 1000 m a.s.l. | 0.5 | 0,2 | 0 |
| Wind loads on buildings | 0.5 | 0,2 | 0 |

# Worked example :

## Determination of Steel Stress

Ratio $g_k/q_k$ = 8.5/4.0 = 2.13



# Worked example :
## Crack Control Without Direct Calculation

From graph $\sigma_{su}$ = 252 MPa

$\sigma_s = (\sigma_{su} A_{s,req}) / (\delta A_{s,prov})$

$\sigma_s = (252 \times 1310)/(1.0 \times 2010)$

= 164 MPa

For H16 @ 100 c/c

Design meets both criteria

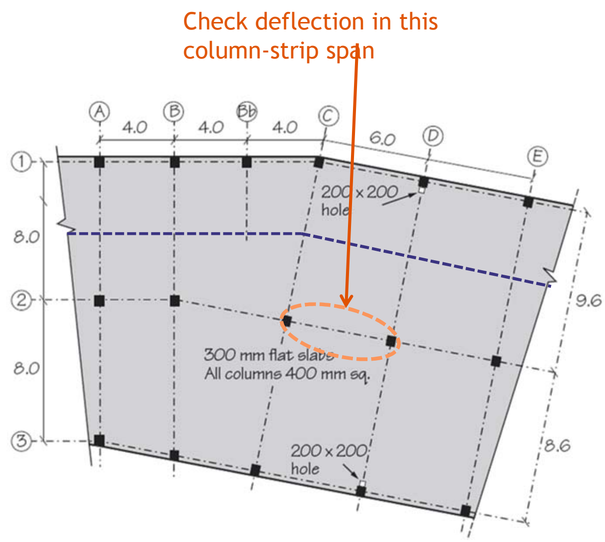
Therefore OK

| *Maximum bar size or spacing to limit crack width* | | | |
|---|---|---|---|
| Steel stress ($\sigma_s$) MPa | $w_{max}$ = 0.3 mm | | |
| | Maximum bar size (mm) | | Maximum bar spacing (mm) |
| 160 | 32 | OR | 300 |
| 200 | 25 | | 250 |
| 240 | 16 | | 200 |
| 280 | 12 | | 150 |
| 320 | 10 | | 100 |
| 360 | 8 | | 50 |
| | For loading or restraint | | For loading only |

# Design exercise

Lecture 6

## Design Exercise:




For the same slab check the strip indicated to verify that:

- deflection is OK and
- the crack widths in the bottom are also limited.

As before:

$A_{s,req}$ = 959 mm$^2$/m B
$d$ = 240 mm
$\gamma_G$ = 1.25
$g_k$ = 8.5 kN/m$^2$
$q_k$ = 4.0 kN/m$^2$
$f_{ck}$ = 30 MPa

## Design exercise: Deflection (pro forma)

Check: basic $l/d$ x F1 x F2 x F3 ≥ actual $l/d$

1. Determine basic $l/d$

   The reinforcement ratio, $\rho = A_{s,req}/bd$ =

   =

## Design exercise: Deflection (pro forma)

Check: basic *l*/*d* x F1 x F2 x F3 ≥ actual *l*/*d*

1. **Determine basic *l*/*d***

   The reinforcement ratio, $\rho = A_{s,req}/bd$ =

   From graph, basic *l*/*d* x *K* = _______ x ____ = _______

**Determine Factor F1**

F1 = ____

For flanged sections where the ratio of the flange breadth to the rib breadth exceeds 3, the values of *l*/*d* given by Expression (7.16) should be multiplied by 0.8.

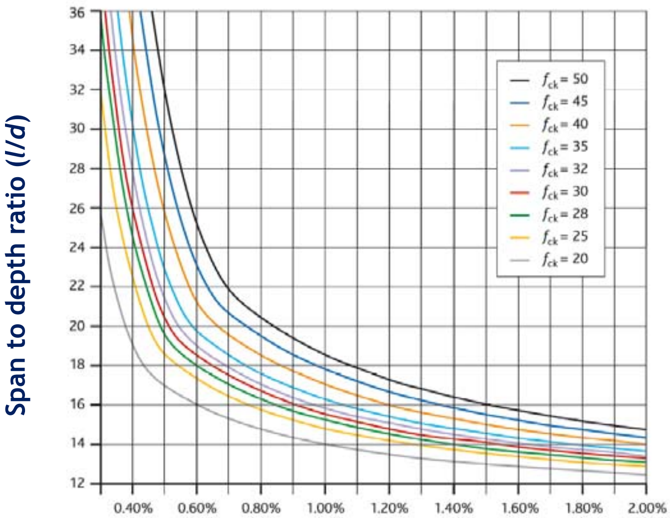
3. **Determine Factor F2**

   F2 = _____

For flat slabs, with spans exceeding 8.5 m, which support partitions liable to be damaged by excessive deflections, the values of l/d given by Expression (7.16) should be multiplied by 8.5 / $l_{eff}$ ($l_{eff}$ in metres, see 5.3.2.2 (1)).

## Basic Span-to-Depth Ratios (for simply supported condition)

How To 3: Figure 5




Percentage of tension reinforcement ($A_{s,req'd}$/bd)

This graph has been produced for *K* = 1.0

| Structural System | *K* |
|---|---|
| Simply supported | 1.0 |
| End span | 1.3 |
| Interior Span | 1.5 |
| Flat Slab | 1.2 |

## Design exercise: Deflection (pro forma)

**4. Determine Factor F3**

$A_{s,req}$ = 959 mm$^2$ (ULS)

Try H___ @ ____ c/c - $A_{s,prov}$ = ____ mm$^2$

F3 = $A_{s,prov}$ / $A_{s,req}$ = _____ / 959 = ____ ≤ 1.5

**5. Check L/d**

Basic l/d x F1 x F2 x F3 ≥ actual span/d

____ x ____ x ____ x ____ ≥ ____ / ____

____ ≥ ____

OK or not OK and revise?

## Design exercise: crack control (pro forma)

Crack Control Without Direct Calculation

From graph for $\gamma_g$ and $g_k/q_k$

(see next slides)

$\sigma_{su}$ = _____ MPa

$\sigma_s = (\sigma_{su} A_{s,req}) / (\delta A_{s,prov})$

$\sigma_s$ = (___ x ___) / (___ x ____)

= _____ MPa

For H__ @ ____ c/c

Does the design meet either criteria?

***Maximum bar size or spacing to limit crack width***

| Steel stress ($\sigma_s$) MPa | $w_{max}$ = 0.3 mm | | |
|---|---|---|---|
| | Maximum bar size (mm) | | Maximum bar spacing (mm) |
| 160 | 32 | OR | 300 |
| 200 | 25 | | 250 |
| 240 | 16 | | 200 |
| 280 | 12 | | 150 |
| 320 | 10 | | 100 |
| 360 | 8 | | 50 |
| | For loading or restraint | | For loading only |

## ψ Factors

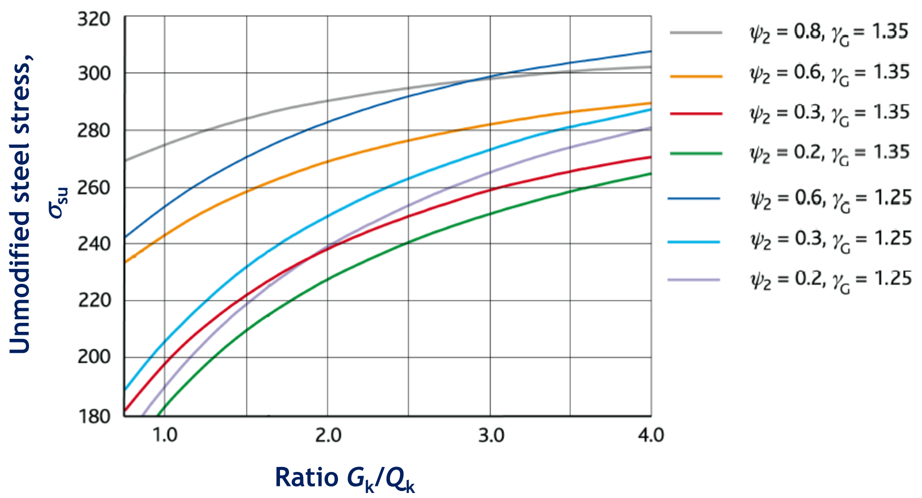
| Action | $\psi_0$ | $\psi_1$ | $\psi_2$ |
|---|---|---|---|
| Imposed loads in buildings, | | | |
| Category A : domestic, residential | 0.7 | 0.5 | 0.3 |
| Category B : office areas | 0.7 | 0.5 | 0.3 |
| Category C : congregation areas | 0.7 | 0.7 | 0.6 |
| Category D : shopping areas | 0.7 | 0.7 | 0.6 |
| Category E : storage areas | 1.0 | 0.9 | 0.8 |
| Category F : traffic area, ≤ 30 kN | 0.7 | 0.7 | 0.6 |
| Category G : traffic area, 30–160 kN | 0.7 | 0.5 | 0.3 |
| Category H : roofs | 0.7 | 0 | 0 |
| Snow load: H ≤ 1000 m a.s.l. | 0.5 | 0.2 | 0 |
| Wind loads on buildings | 0.5 | 0.2 | 0 |

(NB: The slab is for an office)

## Design exercise: Deflection (pro forma)

### Determination of Steel Stress

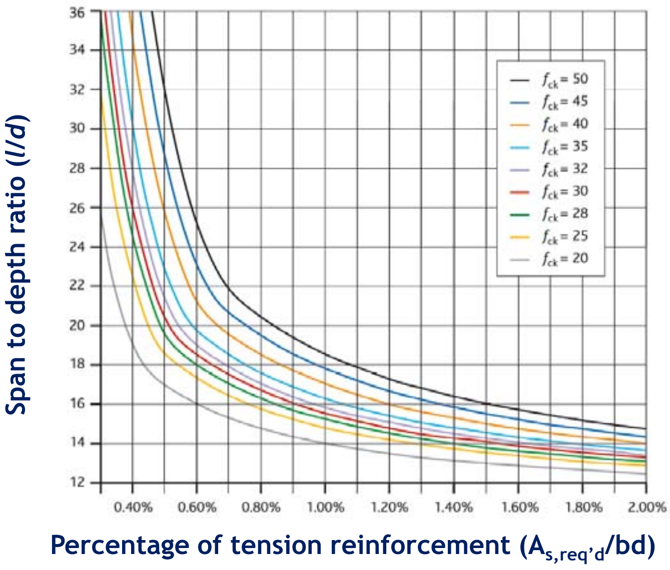
Ratio $G_k/Q_k$ = _____/_____ = _______



## Working space

## Design aid for L/d ratios
### (for simply supported condition)

How To 3: Figure 5




This graph has been produced for $K$ = 1.0

| Structural System | *K* |
|---|---|
| Simply supported | 1.0 |
| End span | 1.3 |
| Interior Span | 1.5 |
| Flat Slab | 1.2 |

## Rebar areas

| Diam ø | Single bar mm² | Spacing 50 | 100 | 125 | 150 | 175 | 200 | 225 | 250 | 275 | 300 | 350 | 400 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | mm²/m | | | | | | | | | | | |
| **6** | 28 | 565 | 283 | 226 | 188 | 162 | 141 | 126 | 113 | 103 | 94 | 81 | 71 |
| **8** | 50 | 1005 | 503 | 402 | 335 | 287 | 251 | 223 | 201 | 183 | 168 | 144 | 126 |
| **10** | 78.5 | 1571 | 785 | 628 | 524 | 449 | 393 | 349 | 314 | 286 | 262 | 224 | 196 |
| **12** | 113 | 2262 | 1131 | 905 | 754 | 646 | 565 | 503 | 452 | 411 | 377 | 323 | 283 |
| **16** | 201 | 4021 | 2011 | 1608 | 1340 | 1149 | 1005 | 894 | 804 | 731 | 670 | 574 | 503 |
| **20** | 314 | 6283 | 3142 | 2513 | 2094 | 1795 | 1571 | 1396 | 1257 | 1142 | 1047 | 898 | 785 |
| **25** | 491 | 9817 | 4909 | 3927 | 3272 | 2805 | 2454 | 2182 | 1963 | 1785 | 1636 | 1402 | 1227 |
| **32** | 804 | 16085 | 8042 | 6434 | 5362 | 4596 | 4021 | 3574 | 3217 | 2925 | 2681 | 2298 | 2011 |
| **40** | 1256 | 25133 | 12566 | 10053 | 8378 | 7181 | 6283 | 5585 | 5027 | 4570 | 4189 | 3590 | 3142 |
| **50** | 1963 | 39270 | 19635 | 15708 | 13090 | 11220 | 9817 | 8727 | 7854 | 7140 | 6545 | 5610 | 4909 |

# End of Lecture 6

- General
- Crack control
- Deflection
- Worked Example
- Design Exercise